Surfaja Royal 1831

THE

# YOUNG MEASURER'S COMPLETE GUIDE;

OR A NEW AND

UNIVERSAL TREATISE

OF

# MENSURATION,

BOTH WITH REGARD TO

THEORY AND PRACTICE.

CONTAINING:

I, A plain and easy introduction to decimal and duodecimal arithmetic, and the extraction of the square and cube roots.

II. A select number of geometrical problems useful in mensuration.

III. The method of measuring all kinds of superficies.

IV. The whole art of surveying or measuring land, explained in the most easy and familiar manner. Also the method of measuring heights and distances.

V. The method of measuring the works of bricklayers, joiners, carpenters, glaziers, painters, plasterers, and plumbers.

VI. The art of measuring solids, both regular and irregular.

VII. The art of gauging all kinds of vessels.

VIII, The practical and true methods of measuring timber, stone, &c.

IX. The manner of calculating tables for measuring timber, &c.

X. The best methods of taking the dimensions of timber, so as to avoid the gross errors attending the common practice.

XI. The art of measuring standing timber.

XII. The manner of finding the tonnage of ships very accurately.

XIII. The method of measuring sawyers work.

XIV. Of measuring cases, &c. sent by sea, and which pay freight by the solid foot.

XV. The method of measuring cord-wood, marle-pits, hay-ricks, statues, and other irregular bodies.

XVI. The construction of the plain scale, and its various uses in the practical branches of the mathematics.

The whole laid down in so easy and familiar a manner, as to render it not only useful for youths at school, but also necessary for artificers of every kind, and for such gentlemen as are desirous of surveying their lands, measuring their buildings, timber, &c.

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The SECOND EDITION, with many Improvements.

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By D. FENNING.

Author of the British Youth's Instructor, or a New and Easy Guide to Practical Arithmetic, and the Royal English Dictionary, &c. &c. &c

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MDCCLXXIX.

## ERRATA.

Page 128, for *Book* II, *read* Book III. Page 257, for *Rule* IV, *read* Rule V.

# THE AUTHOR'S PREFACE.

THE ſeveral books I have publiſhed for the uſe of ſchools, and perſons who have not enjoyed a mathematical education, having met with a kind reception, my friends, particularly thoſe, who devote their whole attention to the inſtruction of youth, have often intreated me to compoſe a treatiſe on the Art of Meaſuring. This requeſt I have accordingly complied with, and hope the work will be as kindly received, as my former publications; eſpecially as the reader will meet with ſeveral particulars here, not to be found in any other book on the ſame ſubject.

Clearneſs and perſpicuity have always regulated my conduct in compoſitions of this kind. I well know, that elegance and conciſeneſs are eſteemed by ſcholars, as the principal beauties in mathematical diſquiſitions; and that, in proportion, as theſe more or leſs abound, the book is ſaid to be more or leſs valuable. But this treatiſe is intended to anſwer a very different purpoſe: It is deſigned to inſtruct thoſe, who are wholly unacquainted with the mathematical ſciences, in the art of menſuration; and in order to accompliſh this valuable end, I have not ſcrupled to ſacriſice both brevity and elegance. Thoſe who have been employed in the inſtruction of youth, know, from painful experience, how difficult it is to make their pupils acquainted with any branch of learning: their inſtructions muſt be often repeated, and even the moſt minute particulars explained, before they can obtain the deſired purpoſe.



In

In the courſe of my teaching, I obſerved, that the beſt books on menſuration required too much attention for youth; that they abounded with difficulties they were unable to ſurmount; and that ſome parts of them could not be underſtood, unleſs the ſcholar had previouſly made a conſiderable progreſs in geometry and algebra, or had recourſe to other writers. Theſe difficulties continually occuring, I determined to draw up a manual for my own uſe, in order to remove theſe obſtructions, and ſave myſelf the trouble of being perpetually engaged in explaining the ſame things.

In order to this, I have begun with a conciſe treatiſe of decimal arithmetic, the extraction of the ſquare and cube roots, and duodecimal arithmetic, or what is commonly called croſs multiplication. Theſe fundamental rules I have endeavoured to lay down in the plaineſt manner; and I hope it will be found that I have not laboured in vain.

The geometrical problems, uſeful in practical menſuration, are next conſidered with all the clearneſs poſſible; and, I dare ſay, may be underſtood by a common capacity, without any other aſſiſtance.

The manner of meaſuring or finding the area of ſuperficies follows: and every difficulty that might obſtruct the learner's progreſs is entirely removed; though at the ſame time the reaſons on which each rule is founded are given, where it was thought neceſſary.

Theſe principles are next applied to practice. The whole art of ſurveying, or meaſuring land is explained, and, I hope, in ſuch a manner, that every perſon may be able, with very little attention, to take the dimenſions of a farm, or other parcel of land, plot it, and give the content in acres, roods, and perches.

The methods made uſe of by artificers, as bricklayers, carpenters, glaziers, joiners, painters, plaſterers, &c. in meaſuring their works, are fully diſplayed by precepts and examples. I have here been very careful to ſhew the learner how he ſhould proceed in taking the dimenſions of theſe reſpective works, in

order

order to guard him againſt errors which might otherwiſe creep into his calculations.

The menſuration of ſolids is next treated of; and I have been careful to inſert the method of meaſuring every ſpecies, neceſſary to be conſidered in a practical treatiſe: for with regard to thoſe of a higher kind, whoſe properties depend upon principles of a more abſtract nature, I have thought proper to omit them entirely, as they are of no uſe in actual menſuration.

From theſe principles the practical rules are deduced, and applied to thoſe arts that are more immediately neceſſary to be known. The methods uſed in gauging all kinds of veſſels are explained in an eaſy and familiar manner; and nothing omitted that is neceſſary to be known in that branch of the mathematics.

In the menſuration of timber, ſtone, &c. I have been very full; becauſe it is an art of the utmoſt importance to be thoroughly known; the common methods being ſubject to ſuch a variety of errors, that unleſs a perſon is well ſkilled in the fundamental principles, he is in danger of ſelling his timber at almoſt one fourth leſs than its real value.

I well know, that the common method of meaſuring by the girt, however falſe it may be in itſelf, is not to be eradicated by reaſon and demonſtration: nor have I attempted it; but I have ſhewn the learner how he may take his dimenſions, even on the common principles, in ſuch a manner, as to avoid the errors that reſult from ignorance and confidence. I have alſo laid down an eaſy method for the conſtruction of tables uſeful in meaſuring timber; ſo that any perſon may eaſily compute a ſet of tables for his own uſe, or examine thoſe inſerted in other books.

I have alſo explained the beſt methods for meaſuring ſtanding timber: and likewiſe how to find the tonnage of ſhips, both by the common rules, and by a very accurate proceſs.

The manner of meaſuring ſawyer's work is next fully explained; the errors attending the common practice pointed out, and a more accurate method ſubſtituted in its room.

To theſe I have added the methods purſued in finding the contents of goods ſent by ſea, which pay freight by the cubic foot; in meaſuring cord-wood, marle-pits, hay-ricks, ſtatues, and other irregular bodies.

Such is the ſubſtance of the following treatiſe; how it may be received by the public muſt be left to time, the great diſcoverer of all events. I have aſſiduouſly laboured to render it a plain, eaſy, and uſeful introduction to the art of meaſuring; and if in endeavouring to execute this intention, I have ſometimes overdone it, and uſed more words and figures than were abſolutely neceſſary, I am ſure the candid and ingenuous will readily forgive me, as the fault flowed entirely from an earneſt ſolicitude to render the work more uſeful to learners.

Errors, no doubt, may be found in this treatiſe; but I hope there are none that will impede the progreſs of the reader. And with regard to the typographical faults of the preſs, I preſume they will be forgiven, as they are common to all books, eſpecially thoſe which treat of mathematical ſubjects.

D. FENNING.

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# EDITOR's

# PREFACE.

THE immature death of the author preventing him from putting the laſt hand to this treatiſe, the papers were ſent to me, with a requeſt that I ſhould reviſe the whole, and complete the parts that remained unfiniſhed. Upon looking over the manuſcript I found, that very few alterations were neceſſary, but that ſeveral of the chapters wanted conſiderable additions; and that ſome particulars were ſtill wanting to render it a complete treatiſe of menſuration. I have therefore ſupplied theſe defects, and endeavoured to imitate the author's ſtyle and manner as near as poſſible. If I have been happy enough to ſucceed, I can venture to ſay, that the book will anſwer the moſt ſanguine expectations of the public; for the rules given by the author are delivered in ſo plain and eaſy a manner, that a perſon even of the meaneſt capacity may eaſily comprehend them, and with very little trouble become an expert meaſurer.

It will alſo be of the utmoſt uſe in ſchools, as it will ſave the maſter the trouble of being perpetually engaged in explaining obſcure paſſages, too often found in other authors; and the ſcholar the painful vexation of ſtudying rules, he is not qualified to comprehend.

Nor are its uſes confined to ſchools; private gentlemen and others, who are deſirous of ſurveying their own eſtates, meaſuring their buildings, their timber, &c. will find this treatiſe a very uſeful companion; as it will remove every difficulty that might otherwiſe obſtruct their deſign, and point out the beſt methods of proceeding in any particular they may deſire to know.

Such youths as have not enjoyed the happineſs of being inſtructed at ſchool in this uſeful branch of

learning,

learning, may, by the aſſiſtance of this treatiſe, and a little cloſe application, obtain, by their own aſſiduity, what their ſituation in life had denied them. And even thoſe, who have for ſome years been engaged in the practical branches of menſuration, will probably find ſeveral particulars in this work, deſerving their attention; and which they may practiſe with credit to themſelves, and advantage to their employers.

I muſt not conclude this preface without acknowledging, that the conſtruction and uſe of the plain ſcale was added at the requeſt of a very ingenious gentleman, who well deſerves the thanks of the public, for the number of ſcholars he has brought up, many of whom are now ſome of the brighteſt ornaments of ſociety.

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# ADVERTISEMENT

## TO THE

## SECOND EDITION.

THE Editor might juſtly be charged with ingratitude, did he not with pleaſure take this opportunity of returning his thanks to the public, for the very favourable reception they have given to the firſt edition of this work. At the ſame time he begs leave to aſſure them, that he has endeavoured ſtill further to merit their good opinion, by rendering this edition more complete than the former, partly by correcting the errors that had unavoidably eſcaped him, partly by inſerting in it, page 97, &c. a more full and accurate method of meaſuring heights and diſtances, than that which was given in the firſt impreſſion.

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THE

# THE INTRODUCTION.

## CHAP. I.

### DIALOGUE I.

*Between* PHILO, *a Master or Tutor, and* DISCIPULUS, *a Scholar; concerning The Rudiments of Decimal Arithmetic.*

*Discipulus.* WILL you permit me, dear Sir, to ask you a favour? All my acquaintance are so pleased with the great progress Tyro and Tyrunculus have made in the most useful parts of learning, by your judicious instructions, that I could not be easy till I came to beg you will be kind enough to take me under your care, and make me acquainted with the art of measuring, which I have for some time desired to understand.

*Philo.* You are sincerely welcome, Discipulus. It gives me the greatest pleasure to see youths desirous of improving their minds, especially in arts so absolutely necessary for the happiness of society. But as I know not, Discipulus, whether you are acquainted with Decimal Arithmetic, the Extraction of the Square and Cube Roots, and the short methods of computation used in measuring superficies and solids; I should be glad to be informed, how far you have proceeded in Arithmetic, that I may lay the proper foundations of

that art, before I proceed to explain its rules; for I hope you are already convinced, that no perſon can be properly ſaid to underſtand any art, till he is acquainted with the principles upon which it is founded.

*Diſc.* I am, Sir, indeed thoroughly perſuaded of the truth of your obſervation; and have always thought, that the true reaſon why the generality of boys, when they come from ſchool, are ſo ignorant, and ſo ſoon forget all they have learned there, is, their having never been thoroughly inſtructed in the fundamental principles. With regard to myſelf, I am entirely ignorant of Decimal Arithmetic, and the other rules you mentioned; and therefore you will begin with the rudiments of the art, and I hope, by attentively liſtening to all your inſtructions, to gain a thorough knowledge of the art of meaſuring, which I have ſo long wiſhed to attain.

*Philo.* There is no doubt, my dear Diſcipulus, of your acquiring, in a ſhort time, this uſeful branch of learning: it depends upon a few principles, eaſy to be underſtood, and extenſive in their application. I ſhall therefore, without any farther preface, proceed to the rudiments of Decimal Arithmetic.

## DIALOGUE II.

### *Of the Nature and Properties of* Decimals, *or* Decimal Fractions.

*Diſcipulus.* WHAT do you mean by Decimals, or Decimal Fractions?

*Philo.* In order to anſwer your queſtion, it will be neceſſary to premiſe, that any thing, conſidered as one, is called an unit, or integer: As one pound, one yard, one gallon, &c. are all units of their reſpective kinds. But, at the ſame time, each may be conſidered as compoſed of ſeveral leſſer quantities; and conſequently, theſe diviſions, whatever they are, muſt be conſidered as parts of their reſpective units. I will endeavour to explain my meaning, by a very familiar example. You very well know that a yard is often divided into four quarters, and contains three feet, or thirty-ſix inches. You

You may therefore consider a yard, though properly an unit or integer, as composed of three feet, or thirty-six inches; or any other number of equal parts, into which it may be supposed to be divided.

*Disc.* I think I understand your meaning sufficiently. You observed, that a gallon, or any other thing, considered as a whole, is an integer. But I know that there are four quarts, or eight pints in a gallon; and therefore the gallon may be considered as composed of four quarts, or eight pints.

*Philo.* You are very right, Discipulus. I am glad you so easily comprehend my meaning; and, for your encouragement, let me add, that I will endeavour to make the whole as easy to you, as this leading principle; attention and a good memory in the scholar, being all that is necessary for attaining any art, when assisted with proper instructions.

*Disc.* You give me great encouragement, Sir, and I will sincerely endeavour to fix all your rules and precepts strongly in my memory. But though I very well comprehend, that any integer or unit may be considered as made up of different parts, yet I do not know what you mean by decimals, or decimal fractions.

*Philo.* You certainly do not; I am now going to explain, what is meant by decimal fractions. The parts, into which any integer or unit may be conceived to be divided, are called the fractions of that integer. Thus one inch is the one thirty-sixth part of a yard, because thirty-six inches make a yard, and is expressed in this manner $\frac{1}{36}$. That is, the number of parts into which the unit, or yard, is supposed to be divided, is placed below the line, and the one, or inch above it.

*Disc.* How are the two numbers that compose the above fraction to be distinguished, if they are called by the same name?

*Philo.* They have different names; that above the line being called the numerator, and that below it, the denominator:

Thus $\left\{ \frac{1}{36} \right.$ Numerator. Denominator.

*Disc.* Is there any reason for giving these numbers different appellations?

*Philo.* Yes. The number placed under the line is called the denominator, becaufe it fhews or denominates the number of parts, into which the unit or integer is divided; while the number above the line is termed the numerator, becaufe it expreffes the number of the fame parts that are contained in the fraction.

*Difc.* I thank you, Sir; I underftand you very well, as I fuppofe thefe names are general, and not confined to this particular fraction.

*Philo.* You are very right, Difcipulus. They are general names; and I will now fhew you how you may always know which is the numerator, and which is the denominator, by this general

RULE.

The denominator is always the divifor; the number of parts into which the integer is naturally divided, is the dividend; the number of the former contained in the latter, is the quotient, and the numerator is the remainder, after the divifion is finifhed.

I will illuftrate this rule by an example. A yard is naturally divided into 36 inches; but fuppofe it were required to divide it into five parts: the work will ftand thus:

```
5) 36 (7
   35
   --
   .1
   --
```

And the anfwer will be 7 inches and $\frac{1}{5}$ (or one-fifth) of an inch.

*Difc.* This is indeed very eafy to be underftood: but fuppofe there be no remainder, will there be no fraction?

*Philo.* There will not, my dear Difcipulus. The quotient then will exprefs the number of parts contained in each of thofe into which the integer is fuppofed to be divided. Thus fuppofe the yard were to be divided into fix equal parts, the work will ftand thus:

```
6) 36 (6
   36
   --
   :
```

And

And as there is no remainder, there is no fraction; but the quotient ſhews, that 6 inches are contained in each of the ſix parts, into which the yard is ſuppoſed to be divided.

*Diſc.* I think I thoroughly underſtand your inſtructions in this particular. If a gallon were ſuppoſed to be divided into 3 parts, each of theſe parts would contain 2 pints and two-thirds of a pint; becauſe the diviſor is 3, the dividend 8, and the remainder 2.

*Philo.* It gives me great pleaſure that you ſo readily comprehend my meaning. But I muſt inform you, that theſe are called vulgar fractions, and are very different from decimals; but it was neceſſary firſt to explain the nature of fractions in general, in order that thoſe of a particular kind may be the more eaſily underſtood.

*Diſc.* Decimals are then only a particular kind of fractions. Are they very different in their nature, and in what does this difference conſiſt?

*Philo.* They are very different from vulgar fractions, and managed with much greater eaſe and expedition: but the difference conſiſts wholly in the denominators, which are always 10, 100, 1000, 10000, &c. that is, the integer or unit is ſuppoſed to be divided into 10 equal parts, and each of thoſe parts into ten others, &c. The integer being thus divided into 10, 100, 1000, 10000, &c. equal parts, they become the denominators to decimal fractions.

*Diſc.* This is very eaſy to be underſtood; but I ſhould be glad to know why they are called Decimals, or decimal fractions.

*Philo.* They have their name from the Latin word *Decem*, which ſignifies *ten*, becauſe the denominators of decimals increaſe in a ten-fold proportion.

*Diſc.* I thank you Sir, for this explanation: the original import of the word tends greatly to fix the idea it conveys more firmly in the memory.

*Philo.* You have made a very juſt obſervation, and you will pleaſe me by often aſking queſtions of this kind, as they might otherwiſe ſlip my memory, and I would not have you ignorant of any thing that can give you ſatisfaction, in the courſe of your ſtudies.

## DIALOGUE III.

### *Of* Notation *of* Decimals.

*Philo.* HAVING, I hope, given you a proper idea of the nature of fractions in general, and the properties of decimals in particular, I ſhall proceed to the Notation of the latter.

*Diſc.* I am greatly obliged to you for your care and attention to remove every difficulty; and will, in return, liſten with the utmoſt attention to every part of your inſtructions.

*Philo.* As the denominators of decimal fractions are always known, it is unneceſſary to ſet them down, and accordingly the numerators only are expreſſed; but they are ſeparated from the whole numbers by a point or comma.

Thus: 5.4, is $5\frac{4}{10}$; 0.7, is $\frac{7}{10}$; 35.05, is $35\frac{05}{100}$, &c.

But perhaps you will conceive a better idea of theſe fractions from the following table, which ſhews the very foundation of decimals; and therefore deſerves to be very attentively conſidered.

| | |
|---|---|
| | &c. |
| 6 | Parts of a Million. |
| 5 | Parts of 100 Thouſands. |
| 4 | Parts of 10 Thouſand. |
| 3 | Parts of a Thouſand. |
| 2 | Parts of a Hundred. |
| 1 | Parts of 10 or $\frac{1}{10}$. |
| 0 | Unit's Place. |
| 1 | Tens. |
| 2 | Hundreds. |
| 3 | Thouſands. |
| 4 | Tens of Thouſands. |
| 5 | Hundreds of Thouſands. |
| | &c. |

*Diſc.*

*Disc.* I will endeavour to understand this table, if you will be kind enough to explain it.

*Philo.* That I will most readily. This table is intended to shew, That, as in the whole numbers, every degree from the unit's place increases towards the left hand, by a ten-fold proportion: so in decimal parts, every degree is decreased towards the right hand by the same proportion, namely by tens.

*Disc.* This appears very plain: and I think I perceive the reason why you called 35.05, where the cypher follows the point of separation, $35.\frac{05}{100}$. because the 5 is by the cypher removed into the decimal parts of a hundred.

*Philo.* You are extremely right; and here you see the reason why cyphers, which placed on the left side of numbers are of no signification, are very different when placed on the left side of decimals. And on the contrary, when cyphers are placed on the right hand of numbers, they increase their value in a ten-fold proportion; but when placed to the right of decimals, they are of no signification. This will be sufficiently explained by a single example. Thus .0002 signifies only two parts of ten thousand; whereas without the cyphers it would have signified 2 tenths. On the contrary, the cyphers placed on the right hand of the whole numbers increase their value in a ten-fold proportion; for 2000, is two thousand; that is, the value of the figure is increased from two to two thousand by the addition of three cyphers: whereas .2000, signifies only two tenths, being of the same value either with or without the cyphers

*Disc.* You are extreamly kind in being so very explicit in your instructions. Is there any thing farther to be observed in the preceding table?

*Philo.* Yes, my dear Discipulus. You may observe, that all whole numbers are in effect nothing more than decimal parts to one another. For in any rank or series of equal numbers, as 666, the second 6 towards the left hand is ten times the value of the next 6 in the last place, or on the right hand of it, and but the tenth part of the 6 on the left. Either of them, therefore, may be taken as integers, or parts of an integer. If integers, they must be set down without a separat-

ing point between them, thus 666. But if they are partly integers, and partly decimals, a ſeparating point muſt be placed between them, as 66.6, or 6.66. In the former the value is 66 and 6 tenths; in the latter, 6 and ſixty-ſix hundred parts of an unit. But they are ſtill in a ten-fold proportion to one another.

From theſe obſervations, compared with the preceding table, it will be eaſy to conceive, that decimal parts take their denomination from the place of the laſt figure: that is, .5 is $\frac{5}{10}$, .56, is $\frac{56}{100}$, .056, is $\frac{56}{1000}$. That .5 is five tenths, .05 is five parts of a hundred, .005, is five parts of a thouſand, .0005, is five parts of ten thouſand, &c.

*Diſc.* I think you have made every particular extremely plain, and I will take care to imprint your inſtructions in my memory.

## DIALOGUE IV.

### Reduction of Decimals.

*Diſcipulus.* WHAT do you mean by Reduction of Decimals?

*Philo.* Reduction is the rule, by which any vulgar fraction is reduced or changed into a decimal of equal value; or the value of a decimal, in the known parts of coin, weight, meaſure, &c. is diſcovered. I ſhall begin with the firſt, *viz.* the method of reducing a vulgar fraction into a decimal of the ſame value, and then proceed to explain the manner of finding the value of any decimal fraction.

### I. To *reduce a* Vulgar Fraction *into a* Decimal.

Thi part of Reduction is performed by the following general

RULE.

Add cyphers at pleaſure to the numerator, and divide by the denominator; the quotient will be the decimal equivalent in value to the given vulgar fraction, obſerving to prick off as many figures in the quotient, as you added cyphers to the numerator.

Ex-

EXAMPLES.

Reduce $\frac{1}{2}$ into a decimal.

2) 10 (5

In the ſame manner $\frac{1}{4}$ will be found equal to .25, $\frac{3}{4}$ equal to .75, and $\frac{5}{8}$ to .625,

Reduce $\frac{9}{56}$ into an equivalent decimal.

```
56) 9.00000000 (.16071428
    56
    ---
    340
    336
    ---
    ..400
      392
      ---
      ..80
        56
        ---
        240
        224
        ---
        .160
         112
         ---
         .480
          448
          ---
          .32
```

Reduce $\frac{6}{17}$ into an equivalent decimal.

```
17) 6.0000 (.3529
    51
    ----
     90
     85
     ----
      50
      34
      ----
      160
      153
      ----
        7
      ----
```

*Disc.* I see no difficulty in performing these operations; but may it not sometimes happen, that there are fewer figures in the quotient, than there are cyphers added to the dividend?

*Philo.* Yes, this is often the case: but whenever it happens, a cypher, or cyphers must be placed on the left hand of the first figure; there being a necessity that the figures, with cyphers if necessarily prefixed, should be equal in number, to the cyphers added to the numerator in the dividend. I will give you an example of this kind.

Let it be required to reduce $\frac{1}{16}$ to an equivalent decimal.

```
16) 1.0000 (.0625
    96
    ----
      40
      32
      ----
       80
       80
      ----
       ..
      ----
```

You obſerve that in the quotient there are only three figures, whereas four cyphers are added in the dividend; I therefore prefix a cypher before the ſix, and the decimal required is .0625.

*Diſc.* I am greatly obliged to you, Sir, and think I now underſtand this rule: but I ſhould be glad to know why there is often a remainder after the diviſion is finiſhed?

*Philo.* The remainder after the diviſion ſhews, that the decimal is not preciſely equal to the given vulgar fraction: but by continuing the diviſion, it may be carried to what degree of accuracy you pleaſe, though, in many caſes, it can never be exact.

*Diſc.* I think I have heard thoſe, who are ſkilled in this part of arithmetic, mention circulating decimals; will you pleaſe to tell me, what they mean by this term?

*Philo.* With all my heart, Diſcipulus. What is called circulating, or repeating decimals, are nothing more than a certain number of figures, which conſtantly return, as long as the diviſion is continued. An example or two will make this extremely plain.

Let it be required, to reduce $\frac{1}{13}$ into an equivalent decimal.

13) 1.000000, &c. (.07692307692307, &c.
91
———
·90
78
———
120
117
———
·30
26
———
·40
39
———
·10, &c.

Here you obſerve, that the figures .076923, are repeated, and would continue to be again repeated if the diviſion were purſued.

Let it be required to reduce $\frac{2}{6}$ into an equivalent decimal.

```
6) 2.000, &c. (.333, &c.
   18 ..
   ——
    20
    18
    ——
     20
     18
     ——
      20
      ——
```

In this inſtance the repetend or circulating decimal is 3, and may be continued to any degree of accuracy deſired.

*Diſc.* This is very ſatisfactory, Sir, and I ſhall always take care, as ſoon as I perceive that the figures, in the quotient, return in the ſame order, not to purſue the work any farther, as I plainly perceive they may be extended without the trouble of dividing.

*Philo.* You are extremely right, Diſcipulus; and it gives me pleaſure to find you are ſo attentive to my inſtructions: I will add, that if you continue the ſame aſſiduity, you will ſoon become a complete meaſurer.

*Diſc.* I ſhould think myſelf undeſerving of your favours, if I did not endeavour to turn them to my own profit. I have indeed often heard the maſter blamed for the ſmall progreſs made by his ſcholars; but am perſuaded, the fault is generally more owing to the inattention of the latter, than to the want of aſſiduity in the former.

*Philo.* I believe this is often the caſe.—But I ſhall now ſhew you how to reduce any known parts of coin, weights, meaſures, &c. into decimal parts. And to make this eaſy, you muſt obſerve the following

RULE.

### RULE.

Reduce the given parts of coin, weight, meaſures, &c. into a vulgar fraction, whoſe denominator is the number of thoſe known parts contained in the integer, and the given parts its numerator.

Let it, for inſtance, be required to reduce 16s. 6d. to the decimal of a pound ſterling?

In order to ſolve this queſtion, you muſt reduce both the numerator and denominator into ſix-pences, becauſe that is the loweſt denomination in the numerator.—Therefore 33 ſix-pences, which are equal to 16s. 6d. make the numerator; and 40, the number of ſix-pences in a pound ſterling, the denominator; and then the queſtion will ſtand thus $\frac{33}{40}$, and the decimal eaſily found as before.

```
40) 33.000 (.825.
    320 · ·
    -------
     ·100
      80
    -------
     ·200
      200
    -------
      ...
    -------
```

Therefore .825, is the decimal required.

### EXAMPLE II.

Let it be required to reduce 13s. 4d. into the decimal of a pound ſterling.

Here the 13s. 4d. muſt be reduced into pence, and will make 160: but the pence in a pound ſterling are 240; and therefore the vulgar fraction will be $\frac{160}{240}$, or $\frac{16}{24}$, and the decimal .6666, &c. as you will eaſily find by dividing the numerator by the denominator.

Ex-

## EXAMPLE. III.

Let it be required to reduce 3 inches to the decimal of a foot.

Here the vulgar fraction is evidently $\frac{3}{12}$, becaufe twelve inches make a foot: and 3, with cyphers annexed, divided by 12, gives .25, the decimal required.

## EXAMPLE IV.

Reduce 7 inches $\frac{3}{4}$ to the decimal of a foot.

Here the numerator will be 31, there being 31 quarters in $7\frac{3}{4}$ inches; and the numerator 48; becaufe 48 quarters of an inch make a foot. Then 31 divided by 48 will give .64583, &c. the decimal required.

```
48) 31.00000 (.64583, &c.
    288 ....
    --------
     ·220
      192
     -----
     ·280
      240
     -----
     ·400
      384
     -----
     ·160
      144
     -----
       16, &c.
     -----
```

I hope, my dear Difcipulus, you have attentively confidered thefe examples, which I think fufficient for the purpofe: but as an exercife for you when alone, I fhall add the following examples with their anfwers, leaving the arithmetical operations to yourfelf.

De-

## Decimals of a Foot.

| | | |
|---|---|---|
| $\frac{1}{4}$ Inch | — is — | .02083. |
| $\frac{1}{2}$ | ——— | .0416. |
| $\frac{3}{4}$ | ——— | .0625. |
| 1 Inch | ——— | .083. |
| 2 Inches | ——— | .16. |
| 3 | ——— | .25. |
| 4 | ——— | .333 &c. |
| 5 | ——— | .416 |
| 6 | ——— | .5. |
| 7 | ——— | .583. |
| 8 | ——— | .666. |
| 9 | ——— | .75. |
| 10 | ——— | .83. |
| 11 | ——— | .916. |

## Decimals of a Pound Sterling.

| | | |
|---|---|---|
| 1 Farthing | — is — | .001 &c. |
| 2 Farthings | ——— | .002. |
| 3 | ——— | .003. |

ALSO,

| | | |
|---|---|---|
| 10*s*. 8*d*. | — is — | .533 &c. |
| 13 : 10$\frac{1}{2}$ | ——— | .69375. |
| 15 : 9$\frac{3}{4}$ | ——— | .790625. |
| 19 : 11$\frac{1}{4}$ | ——— | .996875. |
| 1 : 10$\frac{1}{4}$ | ——— | .0927083. |

Theſe examples being well conſidered, and the operations at large often performed, will render it extremely eaſy for you to reduce any known parts of an integer into decimal parts.

*Diſc.* I will carefully ſtudy theſe examples, and do not doubt of being ſoon maſter of the whole. But I would aſk one queſtion: it is this, whether the numerator and denominator muſt not always be reduced into the loweſt denomination of the former? For inſtance, muſt not the 15*s*. 9*d*. $\frac{3}{4}$. in one of the above examples, be reduced into farthings? If ſo, the vulgar fraction will ſtand thus $\frac{759}{960}$, becauſe 759 farthings make 15*s*. 9$\frac{3}{4}$*d*. and 960 farthings make a pound.

*Philo.* You are extremely right; and it gives me the highest pleasure to find you so readily comprehend my meaning. I shall therefore now proceed to the second part of Reduction of Decimal Fractions, or that of finding their value.

II. *To find the Value of a Decimal in the known Parts of Money, Weights, Measures,* &c.

This part of Reduction is nothing more than the converse of the former, and is performed by the following

RULE.

Multiply the decimal by the next inferior denomination, and prick off, from the product as many figures or cyphers, as there were figures, &c. in the given decimal: multiply the figures so pricked off by the number of figures in the next inferior denomination, and prick off the figures, &c. as before; proceeding in this manner, till you arrive at the least denomination. Then are the figures cut off on the left hand equivalent to the given decimal.

EXAMPLES.

What is the value of 0.825 decimals of a pound sterling?

OPERATION.

| | | |
|---|---|---|
| | 0.825 | |
| | 20 | Shillings in a Pound. |
| Shillings | 16.500 | |
| | 12 | Pence in a Shilling. |
| Pence | 6.000 | |

Answer 16*s.* 6*d.*

Here you observe, that I multiply the given decimal 0.825, by 20, because 20 shillings make a pound, and prick off three decimals in the product; and the answer is 16 shillings, and .500 decimal parts, which being multiplied by 12, the number of pence in a shilling, and

and three decimals pricked off, the anſwer will be ſixpence. Had the decimals remaining been figures inſtead of cyphers, they muſt have been multiplied by 4, the number of farthings in a penny; but as the cyphers are of no value, the queſtion is finiſhed, and 16*s*. 6*d*. the true value of the given decimal.

Let it be required to find the value of 0.7471, decimal parts of a pound.

OPERATION.

```
 0.7471
     20
-------
14.9420
     12
-------
11.3040
      4
-------
 1.2160
```

Anſwer 14*s*. $11\frac{1}{4}$*d*. nearly.

What is the value of 0.74722 decimal parts of a pound, troy weight?

OPERATION.

```
0.74722
     12 Ounces in a pound Troy.
-------
8.96664
     20 Penny-weights in an ounce.
-------
19.33280
     24 Grains in a penny-weight.
-------
 133120
  66560
-------
7.98720
```

Anſwer 8 ounces, 19 penny-wts. 8 grains nearly.

*Diſc*. Why do you ſay 8 grains nearly, when there is only 7 pricked off, in the place of grains?

*Philo*. It gives me pleaſure you have taken notice of it: I intended to have explained the reaſon why I made

made it 8 inſtead of 7 grains in the anſwer. In order to this you muſt obſerve, that though only 7 is ſeparated, there ſtill remains a large decimal, amounting to more than 9 tenths of another grain: conſequently 8 grains is much nearer the truth than 7.

*Diſc.* I thank you, Sir, and find that the real value of a decimal cannot be found, till the decimal, after multiplication, becomes a cypher or cyphers.

*Philo.* Extremely right: and you will obſerve, that in the queſtion preceding the laſt, the value of the given decimal is not accurately found: but as the decimal after the laſt operation is little more than two tenths, or one fifth of a farthing, it is of too little conſequence to be regarded.

*Diſc.* I ſhall very carefully attend to your inſtructions, and ſhould be obliged to you for a few more examples, that I may, by practice, become perfect in this part of decimal arithmetic, as I perceive already, it is of the utmoſt conſequence to be well underſtood.

*Philo.* I will very readily give you other examples; and in order to render them of more advantage, ſhall only give the anſwers.

What is the value of 0.0095 decimal parts of a pound ſterling?

Anſwer, $2\frac{1}{4}$ *d.*

What is the value of 0.0428 of a pound ſterling?

Anſwer, $10\frac{1}{4}$ *d.*

What is the value of 0.87708 of a pound ſterling?

Anſwer, 17*s.* $6\frac{1}{2}$*d.*

What is the value of 0.87628 of a pound Troy?

Anſwer, 10 oz. 10 pwts. 7 grs.

What is the value of 0.798645 of a hundred weight avoirdupoiſe?

Anſwer, 3 qrs. 5 lb. 7 oz.

What is the value of 0.9595 of a foot?

Anſwer, 11 inches and 2 qrs.

DIA-

## DIALOGUE V. SECT. I.

### Addition *of* Decimals.

*Philo.* I Hope, my dear Difcipulus, you are now mafter of the queftions I gave you at our laft meeting, and therefore fhall now proceed to Addition of Decimals, which is performed by the following

RULE.

Set down the numbers propofed to be added, in fuch a manner, that thofe of the fame value may ftand directly under each other, whether they be mixed numbers, or decimal parts; which may be very eafily done, if you place the feparating points in every line under one another. Then, add them as if they were all whole numbers, feparating from the fum as many figures, &c. as there are decimals in the greateft of the given expreffions.

EXAMPLES.

34.5
65.3
128.74
95.0
87.8
7.9

419.24 the fum required.

45.07
50.758
123.0057
75.702
24.8

319.3357 Anfwer.

574.678953
95.79643
78.0546
54.789
8.9

812.218983 Anfwer.

1.957642.

1.975642
.745257
.000598
.8007
.64053

4.162727 Anſwer.

*Diſc.* If all the rules of decimal arithmetic, be attended with no more difficulty than this of Addition, I hope ſoon to be maſter of that uſeful method of computation: for I obſerve, that the only caution neceſſary, is to place ſeparating points directly under one another; the whole ſum being added, as in common arithmetic, without any regard to the ſeparating points, till the operation is finiſhed, and then to ſeparate as many figures towards the right hand, as there are in the greateſt number. Thus in the firſt example, becauſe there are two decimals in the third line, two figures are ſeparated towards the right hand in the total.

*Philo.* You are extremely right, Diſcipulus: and, as there are no exceptions to this general rule, we will proceed to the next in order.

## SECT. II.

### SUBTRACTION.

This, like the former, is performed, in all reſpects, like Subtraction in whole numbers; and the rule reſpecting the points, and the number of figures to be ſeparated in the remainder, are the ſame as before in Addition.

EXAMPLES.

From 74.918765
Take 69.180714

Remains 5.738051

From 8.100007
Take 6.9718743

Remains 1.1281327

From

From 28.007
Take 6.98798012

Remains 21.01901988

From 77.38160007
Take 4.9812

Remains 72.40040007

*Disc.* This is indeed very plain; and I will endeavour to imprint it in my mind by frequent practice.

*Philo.* A very proper resolution, Discipulus; for without practice it is impossible to become an expert arithmetician. Rules without practice are soon forgotten, and errors are perpetually committed by those who are not accustomed to operations.

## SECT. III.

### MULTIPLICATION.

Multiplication of decimals is also performed in the same manner, as it is in whole numbers; but in order to find the true value of the product, you must observe the following

### RULE.

Separate as many figures towards the right hand in the product, as there are decimal parts in both the multiplicand and multiplier added together. But if there are not so many places in the product, the defect must be supplied by cyphers on the left hand. A few examples will sufficiently explain this rule.

### EXAMPLE I.

```
   123.024
     2.232
   -------
    246048
   369072
  246048
 246048
 ---------
 274.589568
```

Here

Here, becauſe there are three decimals in the multiplicand, and three in the multiplier, I ſeparate ſix figures towards the right hand in the product.

EXAMPLE II.

```
      3684.7928
        84.216
    -----------
      221087568
     36847928
    73695856
   147391712
  294783424
  -------------
  310318.5104448
```

EXAMPLE III.

```
     .2365
     .2435
    ------
     11825
     7095
    9460
   4730
   -------
  .05758775
```

In the laſt example you will obſerve, that there are four places of decimals in the multiplicand, and the ſame number in the multiplier, therefore eight places muſt be ſeparated in the product; but as there are but ſeven, a cypher is added on the left hand to make up the number.

*Diſc.* I hope I thoroughly underſtand your meaning; but ſhould be glad to know the reaſon for placing the cypher on the left hand of the product.

*Philo.* The reaſon is extremely plain: any number multiplied by a fraction, or what is leſs than a unit, will be leſs after the operation than before. Thus, if 25 be multiplied by .5 or one half; the product will be only 12.5, or 12 and a half, and if 25 were to be multiplied by .05, the product would be no more than 1.25, or one and a quarter. But in the above example both the multiplier and multiplicand are leſs than unity; and

and consequently they must produce a number much less than unity. This will plainly appear from the following

### Example IV.

```
       .0347
       .0236
     --------
        2082
       1041
       694
     --------
   .00081892
```

Here the places in the product are only five, whereas there are eight decimals in the multiplicand and multiplier; three cyphers are therefore added to supply the defect.

I shall add the following examples as an exercise to these rules, and hope you will carefully perform the operations.

### Examples.

1. Multiply 57.056, by 0.778. Answer 44.389568.
2. Multiply 7.6543, by 5.4246. Ans. 41.52151578.
3. Multiply 0.56879, by 0.05674. Answer 0.0322731446.
4. Multiply 0.03246, by 0.02364. Answer 0.0007673544.
5. Multiply 87649, by 0.03687. Ans. 3231.61863.
6. Multiply 94.35786, by 6.57869. Answer 620.7511100034.
7. Multiply 3.141592, by 52.7438. Answer 165.6995001296.

*Disc.* You are very kind, Sir, and as you have already explained the manner of pricking off the decimals, I shall be very soon able to work the above questions at large. I observe, that in the fifth example, the multiplicand is entirely composed of whole numbers, and the multiplier wholly of decimals. The operation I perceive is however the same, because you have

have ſeparated five places on the right hand, the number of decimals contained in the multiplier.

*Philo.* You have made a very juſt obſervation, Diſcipulus. It gives me great pleaſure to find that you retain the rules ſo firmly in your memory. They are indeed very eaſy, and therefore I have not ſo often repeated them, as I may perhaps think neceſſary hereafter. But you will at preſent obſerve, that in multiplying decimals by decimals, or where there are ſeveral places of decimals, there are often many more decimals than are neceſſary in the product. For in Practice three or four places of decimals are generally ſufficient; and a greater number is troubleſome and unneceſſary. I ſhall therefore ſhew you the proper method of proceeding in caſes of this nature; by which means great part of the labour will be ſaved, and yet the queſtion be anſwered ſufficiently accurate. This uſeful contraction is performed by the following

### RULE.

Set the unit's place of the multiplier directly under that figure in the multiplicand, whoſe place you intend to keep in the product; and invert the order of all its places; that is, write the decimals on the left, and the integers, if any, on the right.

Then, in multiplying, omit all the figures placed, in the multiplicand, to the right of the digit, with which you begin the operation, and let the right hand place of every line ſtand under each other.

But be ſure to remember, that the firſt figure in each line of the operation be increaſed by the carriage, which would ariſe from the places omitted; that is, carrying 1 from 5 to 15; 2 from 15 to 25; 3 from 25 to 35, &c. inſtead of carrying one for every ten, &c. and the ſum of theſe lines will give the product generally exact.

A few examples will make this rule very eaſy, if you take care to imprint it in your memory.

### EXAMPLE I.

Let it be required to multiply 3.141592 by 52.7438; and let there be only four decimal parts in the product.

Theſe

These numbers, according to the above rule, must be placed thus:

| | |
|---|---|
| 3.141592 | The multiplicand, in its natural order. |
| 8347.25 | The multiplier inverted. |
| 1570796 | The prod. by 5, with carriage of 5 times 2. |
| 62832 | The prod. by 2, with carriage of 9 times 2. |
| 21991 | Product by 7, with its proper increase. |
| 1257 | Prod. by 4, &c. |
| 94 | Prod. by 3, &c. |
| 25 | Prod. by 8, &c. |
| 165.6995 | The true product required. |

The reason of this contraction will be very obvious from the whole operation being wrought at large, in the following manner.

| | |
|---|---|
| 3.141592 | |
| 52.7438 | |
| 25 | 132736 |
| 94 | 24776 |
| 1256 | 6368 |
| 21991 | 144 |
| 62831 | 84 |
| 1570796 | 0 |
| 165.6995 | 001296 |

From this example it is evident, that all the figures on the right hand of the line through the operation are omitted in the former contracted method; and that the last single product here, is the first there; though the result is equal in both as far as four decimal places.

You will also observe, that because four decimal places only are to be kept, the multiplier 2, is placed under 5, the fourth place in decimals in the multiplicand.

### EXAMPLE II.

Multiply 384.672158 by 36.83645; and let only four Decimal Places be kept in the Product.

| | |
|---:|:---|
| 384.672158 | Multiplicand. |
| 54638.63 | Multiplier inverted. |
| 115301647 | |
| 23080329 | |
| 3077377 | |
| 115402 | |
| 15387 | |
| 1924 | |
| 14169.2066 | Product required. |

Had this queſtion been ſolved in the common manner, it would have ſtood thus:

| | |
|---:|:---|
| 384.6 | 72158 |
| | 36.8345 |
| 1923 | 360790 |
| 15386 | 88632 |
| 115401 | 6174 |
| 3077377 | 264 |
| 23080329 | 48 |
| 115401617 | 4 |
| 14169.2066 | 038510 |

*Diſcipulus.* I think, Sir, I underſtand your meaning; but if you will pleaſe to work over one or two of the firſt figures in the multiplier, it will perhaps render it ſtill plainer.

*Philo.* With all my heart, Diſcipulus. I ſhall be always ready to remove every difficulty that may in the leaſt retard your progreſs.

We will begin with the firſt line; where I ſay, 3 times 8 is 24, and carry 2; 3 times 5 is 15 and 2 is 17, now ſet down 7 and carry 1, &c. becauſe this is the product ariſing from the multiplication of the 5 that ſtands over the 3, the other being omitted as of no value, but to know what is to be carried: the remaining part of the line is the ſame as in common multiplication.

In the ſecond line I ſay 6 times 8 is 48, and carry 5, becauſe greater than 45, otherwiſe only 4 muſt have been carried; 6 times 5 is 30, and 5 is 35, carry 3; 6 times 1 is 6, and 3 is 9: Now being come to the figure over the 6, ſet down 9.

In

In the third line I say, 8 times 5 is 40, and carry 4; 8 times 1 is 8, and 4 is 12, and carry 1; 8 times 2 is 16, and 1 is 17: now being come to the figure over 8, set down 7, and carry 1, &c. In this manner I proceed with every figure in the inverted multiplier, till the whole work is finished.

*Discipulus*. I am extremely obliged to you, Sir, and shall be able to pursue your instructions without any farther difficulty.

*Philo*. I hope you will, Discipulus; but in order to assist you still farther, I will add a few examples, and where I think the least difficulty occurs, I will add the work at large.

## EXAMPLE III.

Let it be required to multiply 395.3758, by .75642, and reserve only four decimal places in the product.

Contracted Operation.

```
395.3758
 .24657
--------
 2767629
  197688
   23722
    1581
      76
--------
299.0696
```

## EXAMPLE IV.

Multiply 257.386 by 76.48, and reserve only the entire product of Integers.

Contracted Operation.

```
257.386
  84.67
-------
  18015
   1544
    103
     20
-------
  19682
```

Operation at large.

| | |
|---:|:---|
| | 257.386 |
| | 76.48 |
| 20 | 58848 |
| 102 | 9424 |
| 1544 | 136 |
| 18014 | 92 |
| 19682 | .58688 |

## SECT. IV.

### DIVISION *of* DECIMALS.

*Philo.* Having, my dear Diſcipulus, conducted you through the three firſt rules of decimals, I ſhall now proceed to Diviſion, the fourth rule, and indeed the moſt difficult of the whole; but it will ſoon be learned, if attention be not wanting.

*Diſcipulus.* I will promiſe to liſten attentively to your inſtructions, and as you have been ſo indulgent as to remove even the appearance of difficulty, I make no doubt of being ſoon able to comprehend this rule, as well as the others.

*Philo.* I doubt it not, Diſcipulus; nor ſhall the plaineſt inſtructions in my power be wanting for this purpoſe; and in order to this, I ſhall firſt give a general rule for diſcovering the true value of the quotient figures.

### GENERAL RULE.

The places of decimal parts in the diviſor and quotient, being counted together, muſt be always equal in number to thoſe in the dividend.

This rule admits of four caſes.

### CASE I.

When the places of decimals in the diviſor and dividend are equal, the quotient will be whole numbers.

EXAMPLES.

### EXAMPLES.

Divide 295.75 by 8.45.

```
8.45) 295.75 (35 The anſwer.
      2535
      ----
       4225
       4225
      ----
       ....
```

Divide .4368 by .0078.

```
.0078) .4368 (56 The anſwer.
        390.
        ----
        .468
         468
        ----
         ...
```

### CASE II.

When the places of decimal parts in the dividend exceed thoſe in the diviſor, cut off the exceſs for decimal parts in the quotient.

### EXAMPLES.

```
24.3) 780.516 (32.12:
      729 : . .
      ---------
      .515
       486
      ------
       .291
        243
       -----
         486
         486
       -----
         ...
       -----
```

```
436) 34246.056 (78.546.
     3052····
      ·3726
       3488
       ·2380
        2180
        ·2005
         1744
         ·2616
          2616
          ····

.534) .30438 (.57.
      .2670·
       ·3738
        3738
         ···
```

## Case III.

When there are not ſo many places of decimals in the dividend as there are in the diviſor, annex cyphers to the dividend, to render them equal, and the quotient will be whole numbers.

### Examples.

Divide 192.1 by 7.684. And 441 by .7875.

```
7.684) 192.100 (25 the quotient required.
       15368·
       ·38420
        38420
        ·····

.7875) 441.0000 (560
       39375
       ·47250
        47250
        ·····
```

Case

## Case IV.

If after the division is finished, there are not so many figures in the quotient as there ought to be places of decimals by the general rule, supply their defect by prefixing cyphers to it.

### Example I.

Divide 7.25406, by 957

```
957) 7.25406 (.00758.
     6699 ··
     ------
     ·5550
      4785
     ------
      ·7656
       7656
     ------
      ····
```

Here you will observe, that the quotient is 758; but as there are five places of decimals in the dividend, and none in the divisor, five places must be pricked off in the quotient. I therefore prefix two cyphers to the figures in the quotient, and it becomes .00758, the true answer.

### Example II.

Divide .7183241, by 7.2.

```
7.2) .71832410 (.0997672.
     648 ·····
     -----
     ·703
      648
     -----
      ·552
       504
      -----
       ·484
        432
       -----
        ·521
         504
        -----
         ·170
          144
         -----
          ·26
         -----
```

*Discipulus.* You have made this rule very easy by dividing it into four Cases, which would otherwise, I perceive, have been very difficult.

*Philo.* It gives me pleasure to observe, that the method I have taken, has rendered this rule easy to be attained; especially as it is the most difficult part of Decimal Arithmetic. I shall add a few examples, containing all the varieties that can possibly happen in division of decimals, leaving the operations at large as an exercise.

Let it be required to divide 49306.6, by 57.4.
Answer, 859.

| | |
|---|---|
| Divide 493.066, by 574. | Answer, .859. |
| Divide 4930.66, by 5.74. | Answer, 859. |
| Divide 49.3066, by 5.74. | Answer, 8.59. |
| Divide 493.066, by .574. | Answer, 85930. |
| Divide 493.066, by .0574. | Answer, 8590. |
| Divide .493066, by .0574. | Answer, 8.59. |

*Discipulus.* I shall very soon be able to work these examples at large; for I perceive, that the figures in both dividend, divisor, and quotient, are the same in all; and differ only in placing the points.

*Philo.* They are all the same, and intended only to shew the different values of the same figures, when the points are put in different places; and which I hope you will carefully observe.

In treating of Multiplication, I shewed you a method of contracting the work, by using no more figures than were necessary for finding the true value of the product, to any given number of decimal places. I will now shew you how you may contract the work in Division: and in order to this, it will be necessary to observe the following

## Rule.

Let each remainder be a new dividend, and for each of these new dividends, point off one figure from the right hand of the divisor; observing at each multiplication, to have regard to the increase of the figures so cut off, as before in contracted multiplication.

Example

## EXAMPLE I.

The Work contracted.

```
7.9863) 70.2300 (8.7938
 ....   638904
        ------
        ·63396
         55904
        ------
         ·7492
          7187
        ------
          ·305
           239
        ------
           ·66
            64
        ------
             2
        ------
```

The work at large.

```
7.9863) 70.23000000 (8.7938
        638904|...
        ------|---
       ·63396 |0
        55904 |1
       -------|----
        ·7491 |90
         7187 |67
       -------|------
         ·304 |230
          239 |589
       -------|-------
          ·64 |6410
           63 |8904
       -------|-------
            0 |7506
       -------|-------
```

I ſhall give two more examples, but without adding the work at large, thinking it entirely unneceſſary.

### EXAMPLE II.

```
9.365407) 87.076326 (9.297655.
          84288663
          --------
          2787663
          1873081
          -------
           914582
           842886
          -------
            71696
            65558
          -------
             6138
             5619
          -------
              519
              468
          -------
               51
               47
          -------
                4

2.756756) 7414.76717 (2689.67118.
          5513512
          --------
          19012551
          16540536
          --------
          24720157
          22054048
          --------
           2666109
           2481080
          --------
            185029
            165405
          --------
             19624
             19277
          --------
               327
               276
          --------
                51
                28
          --------
                23
                22
          --------
                 1
```

*Discipulus.*

*Discipulus.* I observe, Sir, that in the last example, the whole work is not contracted, but a part only; will you be kind enough to tell me the reason for that particular.

*Philo.* With all my heart: I made use of an example of that kind, that you might observe the difference. When there are more figures than the divisor multiplied by the first quotient figure will extend to, they must be brought down, and the work continued in the common manner; but after this, the work is performed in the same manner as in other examples.

*Discipulus.* You have entirely removed this difficulty, Sir: But I should be glad to know the reason for pricking off the decimals in all these contracted examples; I cannot reconcile it with the general rule.

*Philo.* A very little consideration, and working the operations at large, would have sufficiently removed this difficulty, which will immediately vanish when it is considered, that though there is no figure actually brought down; yet a cypher is supposed to be added to each new dividend. When these are counted with the other decimals in the dividend, you will find the decimals in the quotient to agree exactly with the general rule.

*Discipulus.* I thank you, Sir; and now see evidently the reasons on which all the operations are founded.

## DIALOGUE V. SECT. I.

### *Extraction of the* SQUARE ROOT.

*Philo.* HAVING explained every thing necessary in decimal arithmetic, I shall now proceed to the extraction of the square root, a rule necessary to be known by all who are desirous of knowing the reasons on which many of the operations in practical measuring are founded.

*Discipulus.* Pray what do you mean by extracting the Square Root?

*Philo.* By extracting the Square Root, is meant, the finding such a number, which being multiplied by itself, the Product shall be equal to the given number. Thus suppose 16 the given number, then will the square root

of that number be 4; becauſe 4 multiplied by itſelf will produce 16. In this operation the given number is generally called the reſolvend; and the root is extracted by the following general

## Rule.

1. Make a point over the place of units, and alſo over every ſecond place, counting from unit's, to the left hand, for integers, and to the right hand for decimals; and the root will always conſiſt of as many figures, as there are points over the reſolvend. Each figure thus pointed, and the next to it towards the left hand, is called a period.

2. Seek the greateſt ſquare number in the left hand period, write the root in the quotient, and place the ſquare thereof under the period: ſubtract the ſquare out of the period, and to the remainder bring down the next period as in diviſion, which will form a dividend.

3. Double the root, and place it to the left of the dividend, for a diviſor; ſeek how often the diviſor is contained in the dividend, omitting its right hand place; write the reſult in the quotient, and alſo on the right of the diviſor.

4. Multiply this increaſed diviſor by the laſt quotient figure: ſubtract the product from the dividend, and to the remainder bring down the next period for a new dividend; double the laſt figure in the laſt diviſor, which with the other figure, or figures, will form a new diviſor: divide with the ſame caution as before, and proceed in this manner till all the periods are finiſhed.

A few examples will make this rule very eaſy.

## Example I.

Let it be required to extract the ſquare root out of 572199960721.

This reſolvend being pointed as directed in the rule, will ſtand thus:

```
                        .  .  .  .  .  .
                        572199960721 (756439
                        49, the greatest square in 57.
1 Divisor  145|         ·821
              5|        725
2 Div.     1506|        ·9699
              6|        9036
3 Div.    15124|        ·66396
              4|        60496
4 Div.   151283|        ·590007
              3|        453849
         1512869|       13615821
                        13615821
                        ........
```

Thus is the whole operation finished, and the root required is 756439.

*Discipulus*. You have rendered the rule much plainer than I thought possible to be done by one example: but, as there are a great many figures, and the operation somewhat tedious, it is very possible that some mistakes may be committed: Is there no way to prove the truth of the operation?

*Philo*. Yes, my dear Discipulus, there is a very easy method of proving all operations of this kind. It consists in multiplying the root by itself, and adding the remainder, if any, and the product will give the resolvend. Thus if you multiply by itself 756439, the product, will be 572199960721, the given resolvend.

## Example II.

What is the square root of 1850701.764025?

Operation.

### OPERATION.

```
            . . . . . . . . .
            1850701.764025 (1360.405.
             1
       23| ·85
        3| 69
     266| 1670
       6| 1596
   27204| ··1101.76
       4|   108816
 2720805| ··13604025
              13604025
           .. ...
```

### EXAMPLE III.

What is the ſquare root of 0.06076225 decimal parts.

### OPERATION.

```
       . . . . .
       0.06076225 (0.2465. The root required.
       .04
  44| 207
   4| 176
 486| ·3162
   6| 2916
4925| ·24625
       24625
      . . . . .
```

You will obſerve, my dear Diſcipulus, that in all the above examples, the given reſolvends have been perfect ſquares; and therefore their roots have been extracted without any remainder: but it very often happens, that the reſolvend is not a true ſquare, and when that is the caſe, ſomething will remain after the extraction is finiſhed. Nor can the roots of theſe numbers be ever accurately found, though you may approach infinitely near

near the truth, by annexing as many periods of cyphers as nou pleaſe to the reſolvend, and continuing the operation as before.

## EXAMPLE IV.

Let it be required to extract the ſquare root of 6968.

### OPERATION.

| | |
|---|---|
| | 6968 (83.4745, &c. |
| | 64 |
| 163 | 568 |
| 3 | 489 |
| 1664 | 79.00 |
| 4 | 6656 |
| 16687 | 124400 |
| 7 | 116809 |
| 166944 | 759100 |
| 4 | 667776 |
| 1669485 | 9132400 |
| | 8347425 |
| | 784275, &c. |

And in this manner may the operation be continued as far as you pleaſe; but the true root can never be accurately found.

## EXAMPLE V.

Extract the ſquare root out of 763958207163.

### OPERATION.

### OPERATION.

```
              . . . . . .
              763958207163 (874047.027
              64
       167 |  1239
         7 |  1169
      1744 |  ..7058
         4 |   6976
    174804 |  ..822071
         4 |    699216
   1748087 |    12285563
         7 |    12236609
 174809402 |  ...489540000
         2 |     349618804
1748094047 |    13992119600
                12236658329
                 1755461271, &c.
```

Theſe examples, which are all worked at large, will, I hope, be ſufficient to explain the method of extracting the ſquare root out of any number; and therefore I ſhall only add the following examples, leaving the operations as an exerciſe for you.

### EXAMPLES.

1. What is the ſquare root of 36372961?
   Anſwer 6031.
2. What is the ſquare root of 24681024?
   Anſwer 4968.
3. What is the ſquare root of 1.0609?
   Anſwer 1.03.
4. What is the ſquare root of 911236798.794365?
   Anſwer 30186.699.
5. What is the ſquare root of 2990667969?
   Anſwer 54687.
6. What is the ſquare root of 7?
   Anſwer 2.6457131106.
7. What is the ſquare root of .125? Anſ. .3535.
8. What is the ſquare root of .00715? Anſ. .084.

I hope

I hope you will be very attentive in working the above examples; because it will render you a complete master of this useful rule. I shall now proceed to teach you the manner of extracting the cube root.

## SECT. II.

### Extraction *of the* Cube Root.

*Discipulus.* What do you mean by extraction of the cube root?

*Philo.* By extraction of the cube root, is meant the finding such a number, which being multiplied by itself, and then multiplied by the above product, shall produce the given number. This is performed by the following

### Rule.

1. Make a point over the place of units, and also over every third place, counting to the left for integers, and to the right for decimals; then will there be as many figures in the root, as there are points over the given number.

2. Seek the greatest cube in the first period, or that nearest the left hand; write the root in the quotient, and the cube under the period; subtract the cube from the period; and to the remainder bring down the next period; call it the resolvend, and draw a line under it.

3. Under this resolvend write the triple square of the root, so that units in the latter may stand under the place of hundreds in the former; under this triple square write the triple root removed one place towards the right; and the sum of these two numbers call a divisor, drawing a line under it.

4. Seek how often this divisor may be had in the resolvend, excepting its right hand place, and write the answer in the quotient.

5. Under the divisor write the product of the triple square of the root, by the last quotient figure, setting the units place of this line, under that of tens in the divisor: under this line write the product of the triple root, by the square of the quotient figure, placing this one place beyond the right of the former; and under this line removed one place farther to the right, set down the

the cube of the laſt quotient figure: add theſe three lines together, and call their ſum the ſubtrahend, drawing a line under it.

6. Subtract the ſubtrahend from the reſolvend; to the remainder bring down the next period for a new reſolvend; the diviſor to this muſt be the triple ſquare of all the quotient added to the triple thereof, and proceed in every other reſpect as before.

*Diſcipulus.* This, Sir, is a very long rule, but I will endeavour to learn it with the greateſt expedition.

*Philo.* A little attention will be ſufficient for the purpoſe; eſpecially after I have fully explained it, by a few examples.

## EXAMPLE I.

Let it be required to extract the cube root out of the number 48228544?

```
 .  .  .
48228544 (364
27 The greateſt cube in 48.
-----
21228 Reſolvend.
     { 27  Triple ſquare of the root 3.
Add  {  9  Triple of 3.
-----
279  Diviſor.
     { 162  Triple ſquare of 3, multip. by 6.
Add  { 324  Triple of 3, multip. by ſquare of 6.
     {  216 Cube of 6.
-----
19656 Subtrahend.
-----
1572544 Reſolvend.
     { 3888  Triple ſquare of the root 36.
Add  {  108  Triple of 36.
-----
38988 Diviſor.
     { 15552   Triple ſquare of 36, mult. by 4.
Add  {   1728  Triple of 36, mult. by 4.
     {     64  Cube of 4.
-----
1572544 Subtrahend.
```

In the above example, the given number is a true cube; for when the laſt ſubtrahend is ſubtracted from the

the laſt reſolvend, there will be no remainder: and the work may be proved by multiplying the root into itſelf, and that product again by the root, when the laſt product will be the ſame with the given number. See the work.

```
       364
       364
     -----
      1456
     2184
    1092
    ------
    132496
       364
    ------
    529984
   794976
  397488
  --------
  48228544   Proof.
```

*Diſcipulus*. I am greatly obliged to you, Sir, for performing this operation in ſo plain and inſtructive a manner: It has explained the rule you at firſt laid down in a very clear manner: and if you will indulge me with a few more examples, I make no doubt of being ſoon able to ſolve any queſtion in this rule.

*Philo*. I will readily give you more e amples, and am pleaſed to ſee you are ſo deſirous of overcoming every difficulty that may happen in the courſe of your practice; and be aſſured, that aſſiduity will eaſily ſurmount every thing that may appear at firſt intricate and troubleſome.

## EXAMPLE II.

What is the cube root of 5735339?

```
   . . .
   5735339 (179
   1
   ----
   4735 Resolvend
Add { 3 Triple square of the root 1.
    { 3 Triple of 1.
   ----
   33 Divisor.
Add { 147 Triple of 7, multip. by square of 7.
    { 21 Triple of 7, multip. by the root 1.
    { 343 Cube of 7.
   ----
   3913 Subtrahend.
   822339 The new resolvend.
Add { 867 The triple square of root 17.
    { 51 Triple of 17.
   ----
   8721 Divisor
   7803 Triple of 17, multiplied by 9.
   4131 Triple of 17, multip. by square of 9.
   729 The cube of 9.
   ----
   822339 Subtrahend.
```

These two examples will be sufficient to direct you in the method of proceeding in the operation: but it may be necessary to add several questions with their answers, leaving the operations as an exercise.

1. What is the cube root of 84604519?
   Answer 439.
2. What is the cube root of 1092727?
   Answer 103.
3. What is the cube root of 27054036008?
   Answer 3002.
4. What is the cube root of 219365327791?
   Answer 6031.
5. What is the cube root of 122615327232?
   Answer 4968.

In

In all the above queſtions the given number is a true cube; but when this is not the caſe, you muſt annex three cyphers to the right hand of the remainder, and proceed in the operation as before, and the remaining figures in the root will be decimals; as in the following queſtions.

6. What is the cube root of 25917056?
   Anſwer 295.9
7. What is the cube root of 93759.575070?
   Anſwer 45.42.
8. What is the cube root of 67507824239?
   Anſwer 4071.8.

When the true cube root of any number cannot be found, you muſt obſerve in proving the queſtion, to add the remainder to the cube of the root, and then the ſum will be equal to the number given, if the operation be rightly performed.

## DIALOGUE VI.

*Of* DUODECIMALS; *Or, the art of multiplying* FEET, INCHES, *and* PARTS; *commonly called* CROSS MULTIPLICATION.

*Diſcipulus.* WHY do you call this rule Duodecimals?

*Philo.* Becauſe the unit or integer, is divided into 12 equal parts. And hence this way of computation is chiefly uſed among workmen, in caſting up the contents of ſuperficial and ſolid works, the lineary dimenſions being generally taken in feet, inches and parts:

BECAUSE,

1 Lineary foot is 12 inches.
1 Inch is 8 or 12 Lineary parts.
Alſo 1 Square foot is 144 inches.
1 Square inch is 64 or 144 ſquare parts.
And 1 Cubic foot is 1728 cubic inches.
1 Cubic inch 512, or 1728 cubic parts.

N. B. The

*N. B.* The difference in the parts, arifes from confidering the inch, as divided into 8 or 12 equal parts.

*Difcipulus.* But might not all this trouble be faved, by taking the linear dimenfions in feet, and decimal parts of a foot?

*Philo.* Certainly it might, and the operation performed with much greater eafe and accuracy: But cuftom has prevailed; and the dimenfions are now generally taken in feet, inches, and parts. I fay generally, becaufe many take the dimenfions in feet and decimal parts: but, as this is not commonly practifed, I would not have you ignorant of the ufual method of computation, though the contents of any figure may be more eafily found by decimals.

*Difcipulus.* I thank you, Sir, for giving me fatisfaction in this particular; and am greatly obliged to you for inftructing me in the methods commonly practifed; as I know it is a very difficult tafk to convince fuch perfons, that their method is not the beft hitherto known.

*Philo.* In order to remove every difficulty attending this method of multiplying feet, inches, and parts, it will be neceffary to obferve, that fome workmen take their dimenfions in feet, inches, and half quarters, or eighth parts: others take them in feet, inches and quarters, reckoning every quarter as three parts; fo that thefe actually follow the duodecimal fcale; though they feldom take notice of any other parts of an inch, than 3, 6, or 9, and give and take, as they call it, for the intermediate ones. This being obferved, the operation of multiplying feet, inches, &c. is performed by this general

## RULE.

1. Under the multiplicand write the correfpondent denominations of the multiplier.

2. Multiply each term in the multiplicand, beginning with the loweft, by the feet in the multiplier; placing each refult under its refpective term, and remembering to carry an unit for every 12 from each lower denomination to its next fuperior.

3. Multiply in the fame manner, all the multiplicand, by the inches in the multiplier; and write the

 refult

refult of each term, one place to the right hand of thofe in the multiplicand.

4. Work in the fame manner with the parts in the multiplier, fetting the refult of each term removed two places to the right hand of thofe in the multiplicand.

And the fum of thefe will give the product required.

### Example I.

Let it be required to multiply 10 feet, 4 inches, 5 parts, by 7 feet, 8 inches, 6 parts.

### Operation.

```
 F.   I.   P.
10 :  4 :  5 Multiplicand.
 7 :  8 :  6 Multiplier.
-----------------
72 :  6 : 11
 6 : 10 . 11 : 4
      5 :  2 : 2 : 6
-----------------------
79 : 11 :  0 : 6 : 6 Anfwer:
```

The product would have been the fame, if the work had been begun with the loweft name in the multiplier; obferving to place the refults right.

### Example

```
     10 :  4 :  5 Multiplicand.
      7 :  8 :  6 Multiplier.
     ---------------
    (         5 :  2 : 2 : 6
Add {   6 : 10 : 11 : 4
    (  72 :  6 : 11
     -----------------------
       79 : 11 :  0 : 6 : 6 Anfwer.
```

Here the anfwer is the fame as before; though the operation is begun in the fame manner as in common multiplication.

From this operation carefully obferved, the following rule is eafily deduced.

### Rule.

Feet multiplied by feet, give feet.
Feet multiplied by inches, give inches.

Feet

Feet multiplied by parts, give parts.
Inches multiplied by parts, give parts.
Parts multiplied by parts, give other parts, each of which is the 144th part of an inch.

I have given you the above example merely that you may not be at a loſs how to proceed, if you ſhould have the dimenſions given in feet, inches, and parts: but this is very rarely done, the dimenſions being thought ſufficiently accurate if taken in feet and inches. I ſhall therefore, in the remaining examples confine myſelf to theſe, as being fully ſufficient to anſwer moſt of the purpoſes of practical menſuration.

## EXAMPLE II.

Let it be required to multiply 7 feet, 10 inches, by 6 feet, 4 inches.

### OPERATION.

| | | | | |
|---|---|---|---|---|
| | 7 : | 10 | | Multiplicand. |
| | 6 : | 4 | | Multiplier. |
| Add | 47 : | 0 | | |
| | 2 : | 7 : | 4 | |
| | 49 : | 7 : | 4 | Anſwer. |

Here I begin with the 6 feet, ſaying 6 times 10 is 60, which is juſt 5 feet, (for according to the above rule, feet multiplied by inches give inches) I therefore ſet 0 under the inches, and carry 5; then 6 times 7 is 42, and 5 is 47, which I ſet under feet; becauſe feet multiplied by feet give feet. I then proceed to the 4 inches, ſaying 4 times 10 is 40, 4 and carry 3; I therefore write the 4 in the place of parts, becauſe inches multiplied by inches, give parts: then 4 times 7 is 28, and 3 that I carried is 31 inches, or 2 feet, 7 inches; becauſe feet multiplied by inches, give inches: I therefore place 7 in the column of inches, and 2 in that of feet. Theſe two lines added together give 49 feet, 7 inches, and 4 parts, the anſwer required.

EXAMPLE

### EXAMPLE. III.

What is the product of 12 feet, 9 inches, multiplied by 10 feet, 6 inches?

OPERATION.

| | | | | |
|---|---|---|---|---|
| | 12 : 9 | | | Multiplicand. |
| | 10 : 6 | | | Multiplier. |
| Add { | 127 : 6 | | | |
| | 6 : 4 : 6 | | | |
| | 133 : 10 : 6 | | | Anſwer. |

Here I begin with the 10 feet, ſaying 10 times 9 is 90 inches, or 7 feet 6 inches; I therefore ſet down 6 under inches, and ſay, 10 times 12 is 120, and 7 that I carried is 127, which I ſet down in the column under feet. Then 6 times 9 is 54 parts, or 4 inches 6 parts, I therefore ſet down 6 in place of parts; and 6 times 12 is 72, and 4 that I carried is 76 inches, or 6 feet, 4 inches; I therefore ſet the 4 under the inches, and the 6 under the feet; I then add the two lines together, and the ſum 133 feet, 10 inches, 6 parts, is the anſwer required.

### EXAMPLE IV.

Multiply 11 feet 11 inches, by 9 feet 9 inches.

OPERATION.

| | | |
|---|---|---|
| | 11 : 11 | Multiplicand. |
| | 9 : 9 | Multiplier. |
| Add { | 107 : 3 | |
| | 8 : 11 : 3 | |
| | 116 : 2 : 3 | Anſwer. |

### EXAMPLE V.

Let it be required to multiply 20 feet 8 inches, by 10 feet 7 inches.

OPERATION.

| | | | | |
|---|---|---|---|---|
| | 20 : 8 | | | Multiplicand. |
| | 10 : 7 | | | Multiplier. |
| Add { | 206 : 8 | | | |
| | 12 : 0 : 8 | | | |
| | 218 : 8 : 8 | | | Anſwer. |

Theſe examples wrought at large, will be ſufficient, if attentively conſidered, to explain this rule: I ſhall therefore only add a few queſtions with their anſwers, leaving the operations as an exerciſe.

EXAMPLES *for* PRACTICE.

1. What is the product of 97 feet, 8 inches, multiplied by 8 feet, 9 inches?
Anſwer 854 feet, 7 inches.
2. Multiply 75 feet, 7 inches, by 9 feet, 8 inches.
Anſwer 730 feet, 7 inches, 8 parts.
3. Multiply 7 feet, 9 inches, by 3 feet 6 inches.
Anſwer 27 feet, 1 inch, 6 parts.
4. Multiply 75 feet, 7 inches, by 9 feet, 8 inches.
Anſwer 730 feet, 7 inches, 8 parts.
5. Multiply 87 feet, 5 inches, by 35 feet 8 inches.
Anſwer 3117 feet, 10 inches, 4 parts.
6. Multiply 259 feet, 2 inches, by 48 feet, 11 inches.
Anſwer 12677 feet, 6 inches, 10 parts.
7. Multiply 179 feet, 3 inches, by 58 feet, 10 inches.
Anſwer 10535 feet, 10 inches, 6 parts.
8. Multiply 257 feet, 9 inches, by 39 feet, 11 inches.
Anſwer 10288 feet, 6 inches, 3 parts.

*Diſcipulus.* Your explanations of this rule have been ſo plain, and the ſeveral ſteps neceſſary in the operation are ſo evident, that I ſufficiently underſtand the whole. But I find the operations, unleſs the numbers are ſmall, to be attended with more trouble, than the ſame operations by decimal arithmetic; and therefore the rule ſeems to me of little uſe.

*Philo.* You are extremely right, Diſcipulus; and a rule for taking dimenſions, decimally divided, would be by far the moſt eaſy method of meaſuring. But cuſtom prevails;

prevails; and when the dimenſions are ſmall, this method is attended with very little trouble.

*Diſcipulus*. It gives me the higheſt pleaſure to find that I am not wholly miſtaken, and I could wiſh that the decimal method only was uſed, as it is ſo very natural, and ſo properly adapted to every thing of this kind. And would it not be very eaſy to have a rule decimally divided, ſo that the dimenſions might be taken, without the trouble of reducing the inches, &c. into decimals?

*Philo*. Extremely eaſy. The foot may eaſily be divided into ten equal parts, and each of theſe parts into ten others: for the diviſions will be very little cloſer than thoſe on the common rule, where the foot is divided into twelve equal parts, and each of theſe diviſions into eight ſmaller: conſequently, the common rule is divided into 96 equal parts; ſo that the former has only 4 diviſions more than the latter. I will add a queſtion, where the ſame dimenſions are taken in the common, and in the decimal manner.

Suppoſe the length of a board be 18 feet, 6 inches, and is to be multiplied by 6 feet, 6 inches.

OPERATION by the Common Method.

```
  18 : 6
   6 : 6
 ---------
 111 : 0
   9 : 3 : 0
 -----------
 120 : 3 : 0 Anſwer.
```

If theſe dimenſions had been taken with a decimal ſcale, they would have been 18.5 feet, and 9.5 feet: and the operation would have ſtood thus:

```
  18.5
   6.5
 ------
   925
  1110
 ------
 120.25
```

Or 120 feet, 3 inches; the ſame as before. When the dimenſions are large, this will be found the moſt eaſy

of ſolution; as the mind is leſs burdened, and conſequently leſs ſubject to miſtakes.

I ſhall conclude this INTRODUCTION with adding a few queſtions, wherein feet, inches, and parts, are multiplied by feet, inches, and parts; becauſe I would have you very ready in ſolving queſtions by this, as well as by the decimal method of computation.

EXAMPLES.

1. Multiply 7 feet, 8 inches, 6 parts, by 10 feet, 4 inches, 6 parts.

Anſwer 79 feet, 11 inches, 8 parts, 3 ſecond parts.

2. Multiply 368 : 7 : 5, by 136 : 8 : 4.

Anſwer 50756 : 7 : 10 : 9 : 8.

3. Multiply 7 : 5 : 9, by 3 : 5 : 3.

Anſwer 25 : 8 . 6 : 2 : 3.

THE

# THE WHOLE ART OF MEASURING SUPERFICIES and SOLIDS.

## BOOK I.

### *Of* MEASURING SUPERFICES.

### SECT. I.

### DEFINITIONS.

I. A POINT has no parts, being nothing more than an aſſignable place in any line.

II. A line hath only length, and is bounded by two points.

III. A right or ſtrait line, is that which lies even between theſe points, or is the neareſt diſtance between them.

IV. A Superficies is a figure which hath length and breadth, and is bounded by lines either ſtrait or curve.

V. Parallel lines are thoſe which lie equally diſtant from one another, in all their parts; and therefore if infinitely extended on the ſame plane would never meet.

VI. When one line is inclined towards another, whether they are right, or circular, they will, when extended, meet, and form an angle; and the point where they meet, is called the angular point.

 VII.

VII. When one right line ſtands upon another, ſo as not to incline to either ſide, but makes the angles on each ſide equal; each of theſe angles is called a right angle; and the line ſo ſtanding on the other, is called a perpendicular to that whereon it ſtands.

VIII. All three-ſided figures are called Triangles, but admit of ſeveral diſtinctions: as Equilateral, when the ſides are equal: Iſoſecles, when only two ſides are equal: Scalene, when the three ſides are unequal; and right-angled, when they have one right angle.

IX. All four-ſided figures are called Quadrilaterals, but are divided into Squares, Parallelograms, Rhombus's, and Rhomboides.

A ſquare, is that where all the angles are right, and the ſides equal.

A Parallelogram, or oblong ſquare, is a figure that hath all its angles right, and its oppoſite ſides equal.

A Rhombus, is that which hath its four ſides equal, but no right angle.

X. A circle, is a plane bounded by one curved line, called the circumference, to which all right lines drawn from a certain point within the figure called its center, are equal.

XI. The diameter of a circle, is a right line drawn through the center, terminated at each end by the circumference; and divides the circle into two equal parts; each of which is called a ſemi-circle: half the diameter is called the Radius.

XII. The circumference of every circle is divided into 360 equal parts, called degrees; each degree into 60 equal parts, called minutes; and each minute into 60 equal parts, called ſeconds, &c. Any part of the circumference is called an arch.

XIII. The chord of an arch, is a right line joining the extremities of an arch; and by which the circle is divided into two unequal parts, called Segments.

XIV. A Sector, is a figure comprehended under two Radius's of a circle, and the arch included between thoſe Radius's.

XV. A Polygon is a figure contained under ſeveral ſides; and called a regular Polygon, if the ſides and angles are equal; but if they are not, it is called an irregular Polygon.

A Poly-

A Polygon has different names, according to its number of ſides; *viz.* if it has five ſides, it is called a Pentagon; if ſix, a Hexagon: if ſeven, a Heptagon; if eight, an Octagon; if nine, a Nonagon; if ten, a Decagon; if eleven, an Undecagon; and if twelve, a Duodecagon.

XVI. The Altitude or height of any figure, is the perpendicular let fall from its ſummit to its baſe, or the line on which the figure is ſuppoſed to ſtand.

XVII. The Area of any figure, is the ſuperficial content of it.

## SECT. II.

### *Explanation of the* CHARACTERS *uſed in this* WORK.

*Philo.* In order to ſhorten the work in the ſeveral operations uſed in Meaſuring, a few characters are made uſe of, and therefore I think it neceſſary to explain them here; and beg, my dear Diſcipulus, that you will peruſe them attentively, and make them extremely familiar; for I aſſure you, they will be found very uſeful in all kinds of operations.

*Diſcipulus.* I ſhall take particular care to obſerve your inſtructions, and ſhall never conſider any thing as a taſk, that you ſhall think fit to impoſe upon me.

*Philo.* I do not doubt your attention, and ſhall take care to impoſe nothing, but what you may eaſily underſtand.

+, *More*, the ſign of Addition. Wherever this ſign is found, it implies, that the number, or quantity following the ſign, is to be added to that which precedes it: thus $3+6$, is read 3 *more* 6, and ſignifies, that 6 is to be added to 3.

—, *Leſs*, the ſign of Subtraction, ſignifies that the number following it, is to be ſubtracted from the preceding number: thus $8-5$, is read 8 *leſs* 5, and ſignifies, that 5 is to be taken from 8.

×, *Into*, the ſign of Multiplication, denotes that the numbers between which this ſign is placed, are to be multiplied together: thus $4 \times 8$, is read 4 *into* 8, or 4 multiplied by 8: And $4 \times 5 \times 6 \times 7 \times 8$, implies, that 4 is to be multiplied by 5; that product multiplied by

by 6; that result multiplied by 7; and the last product multiplied by 8. See the work.

$$\begin{array}{r} 4 \\ \underline{5} \\ 20 \\ \underline{6} \\ 120 \\ \underline{7} \\ 840 \\ \underline{8} \\ 6720 \end{array}$$

÷, *By*, the sign of Division, signifies that the number going before the sign, is to be divided by the number following it: thus 20÷5, implies that 20 is to be divided by 5. But the common method now used for expressing division is, by placing the two numbers, in the form of a fraction, the dividend above, and the divisor below it. Thus if 284 were to be divided by 32, it is expressed in the following manner, $\frac{284}{32}$.

=, *Equal*, the sign of Equation, signifies that the numbers, or expressions on each side of it, are equal to one another: thus 6+2=8, or 6 *more* 2 is *equal* to 8. Also 9—3=4+2, or 9 *less* 3 is *equal* to 4 *more* 2: and 8 × 9=72, or 8 multiplied by 9 is *equal* to 72: also $\frac{15}{3}$=5, or 15 divided by 3 is *equal* to 5.

The terms in proportion to one another, are expressed in the following manner: 8 : 12 : : 16 : 24, that is, 8 is to 12, as 16 is to 24. The two points therefore between the 8 and 12, signify *is to*; the four points between the 12 and 16 imply *as*; and the two points between the 16 and 24 denote *is to*.

√, The sign of the square root, implies, that the square root of those numbers or quantities to which it is prefixed, must be extracted: thus √ 81, signifies that the square root of 81, must be extracted. When several numbers or quantities, connected by the other signs or characters, are to have their square root extracted, a line is drawn over them, to denote what part of the expression is to have its square root extracted; as $\sqrt{16 \times 14}$, implies that 16 is to be multiplied by 14,

and

and the ſquare root is to be extracted out of the product.

The character $\sqrt[3]{}$ denotes the cube root of the quantities following it: thus, $\sqrt[3]{397}$, implies the cube root of 397: and $\sqrt[3]{32 \times 69}$, ſignifies, that 32 is to be multiplied by 69, and the cube root extracted out of the product.

When ſeveral terms, or numbers, are connected by lines drawn over them, it denotes the reſult of thoſe terms or numbers, when the operations denoted by the ſigns are performed: thus $\overline{6-4}\times\overline{6+4}$, implies, that 6 *leſs* 4, or 2 is to be multiplied by 6 *more* 4, or 10: and $8 \times \overline{6-4+14}$, ſignifies that 8 is to be multiplied by 6 *leſs* 4 *more* 14, or 16. But had the line been extended only to the firſt 4, or $8 \times \overline{6-4}+14$, the expreſſion would have been very different; for it would then have ſignified, that 8 was to be multiplied by 6 *leſs* 4, or 2, and 14 added to the product: in the firſt caſe, the reſult would have been 124, and in the latter only 30.

Hence appears the neceſſity of being careful in drawing theſe lines; as they greatly alter the value of the expreſſion.

Theſe, my dear Diſcipulus, are all the characters neceſſary for you to uſe in any kind of menſuration: and I would adviſe you to get them perfectly by heart, and practiſe them frequently, by which means they will ſoon become as familiar to you, as the common figures.

*Diſcipulus.* It ſhall be my buſineſs to imprint them firmly in my memory. Indeed their uſe in ſhortening the operations is ſo apparent, that I ſhould think I profited very little by your inſtructions, if I neglected to learn what has ſo great a tendency to promote my own advantage.

*Philo.* It gives me pleaſure to hear you make ſuch juſt obſervations: and I ſincerely wiſh all ſcholars would endeavour to imitate your example; this would at once give ſatisfaction to their maſter, and delight to their parents. Give me alſo leave to point out to you another inſtance, where theſe ſigns are very neceſſary: I mean in expreſſing the operations proper to be performed, when you are not in a place convenient for the

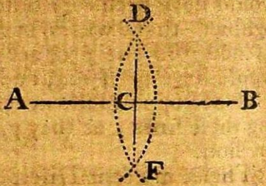
purpose. The operations in measuring are generally long, and require a considerable degree of care and attention; and therefore often very improper to be performed on the spot where the dimensions are taken. But the whole may be easily expressed by the assistance of these characters, and you will thence be enabled to perform the requisite operations at your leisure. But in order to reap this advantage, it will be proper for you to be very ready in expressing any calculation necessary to be performed; and this you will soon acquire by practice.

## SECT. III.

### *Of such* GEOMETRICAL PROBLEMS, *as are necessary in* MENSURATION.

### PROB. I.

*To divide a right line, as* AB, *into two equal parts.*

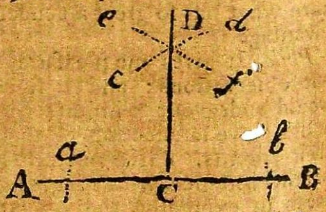



From A and B, the ends of the given line, and any Radius greater than one half of AB, describe two arches of circles crossing each other in two points, as D and F, then join those two points D and F with a right line, and it will bissect, or divide the line AB, into two equal parts, as at C.

### PROB. II.

*To raise a perpendicular on a given right line, from a point assigned, situated in, or near the middle of it.*

Let the given line be AB, and let it be required to erect a perpendicular to it, in the point C.

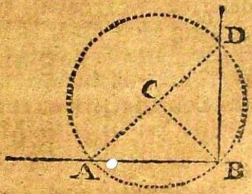



Place one point of your compasses in the point C, and open them to any convenient distance, as to *a*, and make a mark at *b* with the same extent of the compasses. Then placing one point of the compasses in *b*, with any convenient Radius greater than *b* C, describe

the arch *c d*, and with the ſame Radius, placing the point in *a*, deſcribe the arch *e f*. Draw the right line CD, from the given point C, through the interſection of the two arches *c d*, *e f*, and it will be the perpendicular required.

## PROB. III.

*To raiſe a perpendicular upon the end of any given line.*

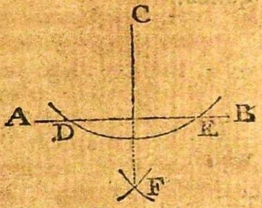



Let the given line be AB, and let it be required to raiſe a perpendicular on the point B. Upon any point at random, out of the line A B, as at C, place one foot of your compaſſes, and extend the other to B, and on C, as a center deſcribe a circle, and from the point where the circle cuts the given line, as at A, draw the circle's diameter ACD: then from the point D, draw the right line D B, and it will be the perpendicular required.

## PROB. IV.

*To let fall a perpendicular upon any right line, from any aſſigned point, that is not in the given line.*

Let the given point be C, and the given line on which the perpendicular is to let fall be A B.

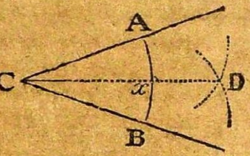



Set one foot of the compaſſes in the given point C, and deſcribe ſuch an arch of a circle as will croſs the given line A B, in two points as D and E. Then ſetting one foot in D open the compaſſes to any diſtance greater than half D E, and deſcribe an arch as at F; and with the ſame Radius, ſetting one point in E, with the other deſcribe another arch, cutting the former in F: Then laying a ruler on F and C, draw the right line C F, which will be the perpendicular deſired.

## PROB. V.

*To biſſect an angle into two equal angles.*

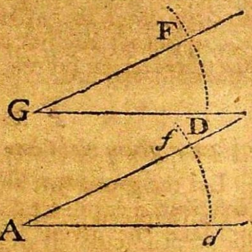
Upon the angular point C as a center, and with any convenient Radius, deſcribe an arch as A B, and from theſe two points A and B, deſcribe two equal arches crоſſing each other as at D. Then join the points C and D, with a right line, and it will biſſect the arch A B, and conſequently the angle, as was required.




## PROB. VI.

*From a point* A *in a given right line, to make a right lined angle, equal to another right lined angle given.*

Upon the given angular point C, and with any convenient Radius, deſcribe an arch, as F D; and with the ſame Radius, ſetting one point of the compaſſes in A, deſcribe the arch *f d*; make *f d* equal to F D; then join the points A and *f*, with a right line, and it will form the angle required.

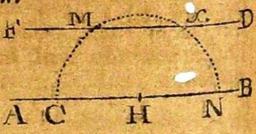



## PROB. VII.

*To draw a right line, that ſhall be parallel to a given right line, and paſs through any aſſigned point.*

Let the given right line, be A B, and the point thro' which the parallel is to paſs be *x*.

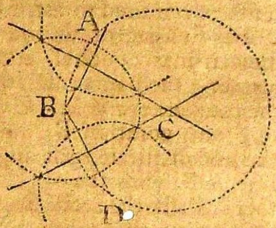
Take any point in the given line as at H, the farther from *x* the better; make H *x* Radius, and with it upon the point H, deſcribe a ſemi-circle, as C M *x* N; then make the arch C M equal to the arch N *x*, and through the points



points M and *x*, draw the right line F D, which will be parallel to A B, as required.

## PROB. VIII.

*To describe a circle that shall pass through three given points, provided they are not situated in a strait line.*

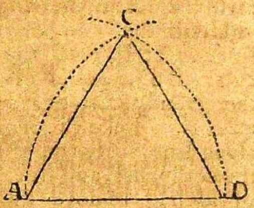



Let the given points be A, B, and D, thro' which it is required the circumference of the circle shall pass. Join the points B A, and B D with right lines, and bissect them both, as already described, Prob. 1. The point where these bissecting lines meet, as at C, will be the center of the circle required; for if you place one of the points of the compasses in C, and extend the other to any of the given points A, B, or D, and with that Radius describe a circle it will pass thro' the three given points.

## PROB. IX.

*Upon a given right line to describe an equilateral triangle.*

Let the given line be AD, on which it is required to describe an equilateral triangle.



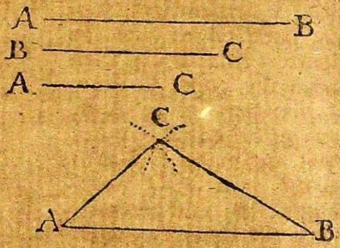

Make the given line Radius, and with that distance place one of the points of the compasses on A, and with the other describe the arch C D: remove the point of the compasses to D, and with the same Radius or distance describe the arch A C; from the intersection of these arches draw the right lines A C, and D C, and you will have the equilateral triangle required.

## PROB. X.

*Three right lines being given, provided any two of them taken together be longer than the third, to form them into a triangle.*

Let the three given lines be A B, B C, and A C, and let it be required to form them into a triangle.

Take either of the shorter lines, as B C, between the points of the compasses, and setting one point in B, the end of the longest line A B, with the other describe the arch; then take the other line, A C, between the points of the compasses, and setting one foot in A, with the other describe a second arch, cutting the former in C; join the points C A, and C B, with right lines, and you will form the triangle required.

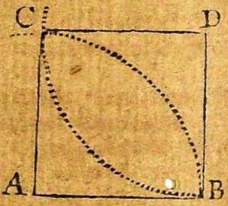



## PROB. XI.

*Upon a given right line to form a square.*

Let the given line be A B, and let it be required to form a square upon it.

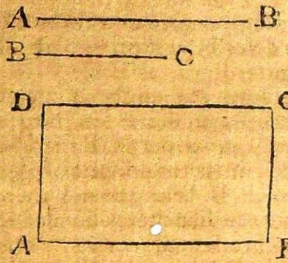
Upon the end of the given line, as at B, erect the perpendicular B D, equal in length to the given line; that is, make B D = A B. Take the length of the given line, between the points of the compasses, and from A describe an arch; remove the point to D, and with the same distance, or Radius, describe another arch cutting the former in C: join the points C A, and C D, with right lines, and they will form the square required.



## PROB. XII.

*Two right lines being given to form of them a right-angled Parallelogram.*

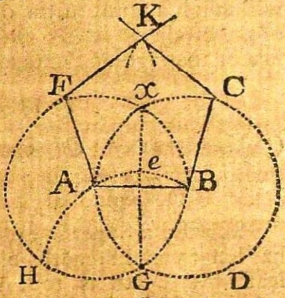
Let the two lines be A B and B C, and let it be required to form of them a right-angled Parallelogram.




Upon an end of the longest line, as at B, erect a perpendicular of the same length with the shorter line B C; then from the point C draw a line parallel to A B, and of an equal length; that is, make D C = to A B; join A D with a right line, and you will form the Parallelogram required.

## PROB. XIII.

*Upon a right line to describe any regular Pentagon, or five sided Figure.*



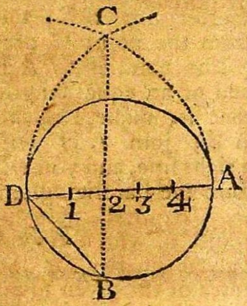

Let the right line be AB, and let it be required to describe on it a regular Pentagon. Make the given line A B Radius, that is, take the length of the line A B, between the points of your compasses, and upon each end of it describe a circle, and upon these points where the circles cross each other, as at G, and $x$, draw the right line G $e$ $x$. Upon the point G, with the same Radius, describe the arch H A $e$ B D; lay a ruler on the points D, $e$, and mark where it crosses the other circle, as at F. Again, lay the ruler upon the points H, $e$, and mark where it crosses the other circle, as at C. Then from the points F and C, with the same Radius as before, describe cross arches, as at K: join the points A F, F K, K C, and C B, with right lines, and they will

form the Pentagon required. For AB=AF=FK=KC=CB; and the angles at A, B, C, K, F, will be equal.

## PROB. XIV.

*In any given Circle to inſcribe any regular Polygon.*

Let the given circle be DAB, and let it be required to deſcribe within it a regular Pentagon, or five ſided figure.




Draw the circle's diameter DA, and divide it into as many equal parts, as there are ſides in the required Polygon, which is here 5; and therefore the diameter is divided into five equal parts. Then make the whole diameter Radius, and deſcribe two arches, as CA, and CD. If a right line be drawn from the point C through the ſecond diviſion on the diameter, as at 2, it will aſſign a point in the oppoſite part of the periphery of the circle as at B; and the line DB, will be one ſide of the regular Pentagon required.

## CHAP. II.

*Of* MEASURING: *Or, finding the Area of* SUPERFICIES.

*Diſcipulus.* WHAT do you mean by a Superficies?

*Philo.* By a Superficies we underſtand a figure, which has length and breadth, but is without thickneſs: at leaſt the depth is too ſmall to be regarded in this Species of Menſuration: the ſurface only being here conſidered.

## SECT. I.

*Of Meaſuring a* SQUARE.

A ſquare has already been defined, in the firſt ſection of the preceding chapter, and the area of it is found by the following

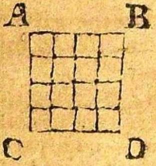
RULE.

### RULE.

Multiply the side into itself, and the product will be the area, or content required.

### EXAMPLE.

Suppose the figure A B C D a geometrical square, whose side, for they are all equal, is 4 feet; and its area or content be required.




### OPERATION.

4
4
———
16 Answer 16 feet.

You will observe, my dear Discipulus, that nothing more is required, than to find how many small squares the sides of which are each one foot, are contained in the given square; for the number of these squares is the area of the given square, as you will easily perceive by examining the annexed figure, and comparing it with the above rule and operation.

## SECT. II.

### *Of Measuring a* PARALLELOGRAM.

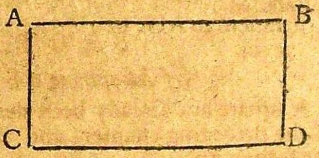
A Parallelogram, is a long square, whose opposite sides are equal and parallel; and its area found by the following

### RULE.

Multiply the length by the breadth, and the product will be the area required.

### EXAMPLE.

Let ABCD represent a Parallelogram, whose length AB, or CD, is 9 feet 6 inches, and the breadth AC, or BD, 6 feet 3 inches; and it be required to find the area.




Opera-

Operation by Decimals.

```
   9.5
   6.25
 ------
   475
  190
 570
 ------
 59.375
     12
 ------
  4.500
      2
 ------
  1.000
```

Anſwer 59 feet, 4 inches and a half.

Operation by Duodecimals, or Croſs Multiplication.

```
  9.6
  6.3
 -----
 57.0
  2.4.6
 -----
 59.4.6
```

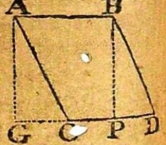
Anſwer 59 feet, 4 inches, and 6 twelfths, or one half, as before.

## SECT. III.

### *Of Meaſuring a* RHOMBUS.

A Rhombus, or Diamond-like Figure, is that which has four equal ſides, but no right angle, and is meaſured by multiplying one of the ſides by a perpendicular, let fall from one of the obtuſe angles upon the oppoſite ſide.

EXAMPLE.




Let ABDC, be a Rhombus whoſe area is required; and let the length AB be 20 feet, 8 inches, and the perpendicular AG, or BP, be 17 feet, 3 inches.

Operation by Decimals.

```
   20.75 Length
   17.25 Perpendicular.
  ------
   10375
   4150
  14525
  2075
  --------
  357.9375
        12
  --------
   11.2500
         8
  --------
    2.0000
```

## SECT. IV.

### *Of Measuring a* RHOMBOIDES.

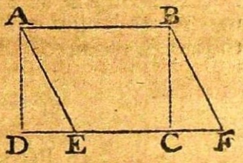
A Rhomboides, is a figure whose opposite sides and opposite angles are equal, but it has no right angle; and is measured by the following

RULE.

Multiply one of the longest sides, by the perpendicular, let fall from one of the obtuse angles, and the product is the content required.

EXAMPLE.

Let ABFE, be a Rhomboides, its length AB=EF, be 20 feet, the perpendicular AD=BC, 12 feet, 6 inches, and let the area be required,




Operation by Decimals.

```
    20
  12.5
  ----
   100
   40
  20
  -----
  250.0
```

Answer 250 feet.

Operation

Operation by Duodecimals.

```
  20
  12—6
 -----
 240
  10:0
 -----
 250  0
```

Anſwer 250 feet, as before.

## SECT. V.

### *Of Meaſuring a* TRIANGLE.

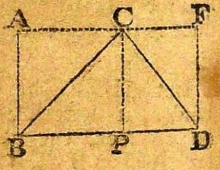
There are various ſorts of triangles, but the area of them all are found by this general

### RULE.

Multiply half the baſe by the perpendicular, or the whole baſe by half the perpendicular; and the product will be the area required.

### EXAMPLE.

Let BCD, be a triangle. From the point C, let fall the line CP, perpendicular to the ſide BD. Let the baſe BD be 40 feet, and the perpendicular CP 20 feet: the area is required.




OPERATION.

```
 20 Half the Baſe.
 20 Perpendicular.
----
400
```

Anſwer 400 feet.

The figure annexed will ſufficiently explain the reaſons on which the rule is founded: for the triangle BCD is half the Parallelogram AFDB. Therefore the area of the triangle BCD, muſt be equal to half the area of the Parallelogram; but the area of the Parallelogram, is equal to BD × CP = 40×20=800; half of which

which is 400 feet, the area of the triangle BCD; and the ſame as before.

But the area of a triangle, when the three ſides are given, may be found without letting fall a perpendicular, by the following

## RULE.

From half the ſum of the three ſides, ſubtract each ſide ſeparately; let the half ſum, and the three differences be multiplied continually, and the ſquare root of the product will be the content required.

## EXAMPLE.

Suppoſe the area of a triangle was required, whoſe ſides were 20, 18, and 16 feet.

## OPERATION.

The ſum of the three ſides 20, 18, and 16, is 54, whoſe half is 27.

And 27—20=7; 27—18=9; 27—16=11. Then

```
       27
        7
    -----
      189
        9
    -----
     1701
       11
    -----
    18711

       . . .
       18711 (136.78
       1
      ----
   23 | 87
    3 | 69
   ------------
  266 | 1811
    6 | 1596
  -------------
 2727 | 21500
    7 | 19089
 --------------
27348 | 241100
    8 | 218784
 ---------------
          22316
```

Anſwer 136.78 feet.

SECT

## SECT. VI.

### *Of Meaſuring a* TRAPEZIUM.

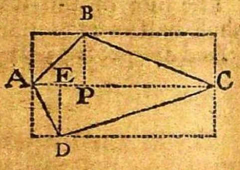
A Trapezium is a quadrangular figure, whoſe oppoſite ſides are not parallel; and its area, or ſuperficial content, is found by the following

### RULE.

Divide the Trapezium into two triangles, by drawing a Diagonal through the figure; let fall perpendiculars from the oppoſite obtuſe angles; take the half of the ſum of the two perpendiculars, and multiply the Diagonal by that half ſum, the product will be the area required.

### EXAMPLE.

Let ABCD be a Trapezium, whoſe area, or ſuperficial content, is required; ſuppoſing the Diagonal AC to be 200 feet, the perpendicular BP 50 feet, and the perpendicular DE, 40 feet.




### OPERATION.

BP 50 + DE 40 = 90, the half of which is 45.

Then 200 = The Diagonal A C.
45 = Half the perpendiculars.

1000
800

9000 Feet, the area of the Trapezium ABCD, required.

N. B. The area would have been the ſame, if the area of each triangle had been found ſeparate, and their areas added together.

200 The Diagonal.
25 Half BP.

1000
400

5000 Area of ABC.

200 The Diagonal.
20 Half DE.

4000 Area of ADC.

Then 5000+4000=9000, the area of the whole Trapezium ABCD, the same as before.

## SECT. VI.

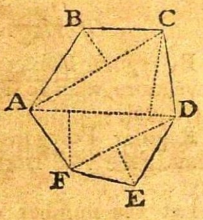
### *Of Measuring irregular* FIGURES.

Irregular Figures are such as have more sides than four, and both the sides and angles unequal; their area, therefore, is found by the following

### RULE.

Divide the irregular figures into Trapeziums and triangles; find the area of each severally, and add all the areas together: the sum will be the area of the irregular figure.

### EXAMPLE.




Let ABCDEF, be an irregular figure, whose area is required. Draw the lines AC, AD, and DF, which will divide the figure into four triangles; and let fall the perpendiculars from the angles, at B, C, E, and F: and let AC be=40, AD=30, and DF=26 feet; and the perpendiculars B=14, C=28, E=15. F=16.

OPERATION.

OPERATION.

| | |
|---|---|
| 40 | = AC. |
| 7 | = Half perp. B. |
| 280 | = Area of ABC. |
| | |
| 30 | = AD |
| 14 | = Half perp. C. |
| 420 | = Area of ACD. |
| | |
| 30 | = AD. |
| 8 | = Half perp. F. |
| 240 | = Area of ADF. |
| | |
| 26 | = DF. |
| 7.5 | = Half perp. E. |
| 130 | |
| 182 | |
| 195.0 | = Area of DEF. |

| | | |
|---|---|---|
| Then | 280 | = Area of ABC. |
| | 420 | = Area of ACD. |
| | 240 | = Area of ADF. |
| | 195 | = Area of DEF. |
| | 1135 | = The Area of ABCDEF. |

## SECT. VII.

### *Of Meaſuring regular* POLYGONS.

By regular Polygons, are meant all ſuch figures, as have more than four ſides, and whoſe ſides and angles are equal.

Regular Polygons are diſtinguiſhed by particular names, derived from the Greek, as is explained in page 3.

Let the number of ſides and angles be what they will, the area or ſuperficial content is found by the following

RULE.

### RULE.

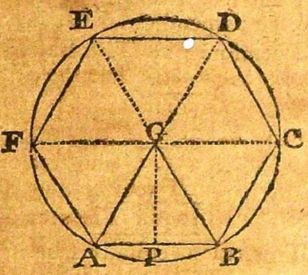
Multiply the ſum of the ſides by half the perpendicular let fall from the center to the middle of one of the ſides: or, multiply half the ſum of the ſides by the perpendicular, and the product will be the content.

### EXAMPLE.

Let it be required to find the area of a regular Hexagon, whoſe ſide is twenty-five yards, and the perpendicular 17.2 yards.

### SOLUTION.

The figure annexed, repreſents a regular Hexagon, or ſix ſided figure inſcribed in a circle; the ſix ſides are the ſtrait lines AB, BC, CD, DE, EF, and FA. And GP, is the perpendicular let fall from the center G, upon the middle of the ſide AB.




Then 25×6=150= to the ſum of the ſides; and half the perpendicular=8.6. Therefore

```
   150
   8.6
  ----
   900
  1200
  ------
1290.0 The area required.
```

```
Or, 75   Half the ſum of the ſides,
   17.2  The perpendicular.
  -----
   150
   525
   75
  ------
  1290.0 The area required.
```

Or, 150 The ſum of the ſides.
17.2 The Perpendicular.

300
1050
150

2|25800|1290.0 The area required.

It is ſufficiently evident from the above figure, that the Hexagon conſiſts of ſix triangles, all equal to one another. If therefore the area of one of the triangles be found, and that area be multiplied by the number of triangles, we ſhall have the area of the whole figure.

Thus in the above figure, the ſide AB is 25 yards, and the perpendicular GP 17.2 yards;

Therefore 25 The Baſe.
8.6 Half the perpendicular.

15.0
200

215.0 The area of the triangle AGB.
6 The number of triangles.

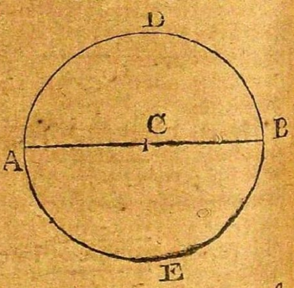
1290.0 The area of the Hexagon, as before.

This rule is equally true in all regular Polygons.

## SECT. VIII.

### *Of Meaſuring* CIRCLES, *and their* PARTS.

A circle is a plane figure, comprehended under one line only, called the circumference, or Periphery.




In the annexed figure, ADBE is the circumference of the circle ABCDE. C, the center, and AB the diameter, half of which, viz. AC, or CB, is called the Radius.

The circle is the moſt capacious figure, or contains more ſpace than any other plane figure of equal compaſs: and at the ſame time, affords a greater number of uſeful properties, moſt of

of which depend upon knowing the proportion, which the diameter bears to the circumference.

It is ſufficiently evident from what we have obſerved in the preceding ſection, that a circle may be conſidered as a Polygon, of an infinite number of ſides; that the Radius, or ſemi-diameter, muſt be equal to the perpendicular of ſuch a Polygon; and that the circumference muſt be equal to the Periphery of the Polygon.

But the labour and difficulty attending this method of finding the ſides of ſuch a Polygon, has induced Mathematicians to attempt the ſolution of this uſeful Problem, by more eaſy and expeditious methods, and they have ſo well ſucceeded, that the calculation is attended with very little difficulty; but the principles on which the modern methods are founded, are of too ſubtile a nature, to be explained in this work: and therefore, my dear Diſcipulus, you muſt be ſatisfied with knowing the foundation, on which the ancients proceeded. Perhaps hereafter, the modern improvements may be explained.

The famous VAN CULLEN, by purſuing the ancient method, found, with infinite labour, the circle's circumference to 36 places of figures, viz. if the diameter of a circle be 1, the circumference will be 3.14159265358979323846264338327950288. Theſe figures were ſaid to be engraven on his tomb-ſtone, in St. Peter's church, at Leyden, as a memorial of ſo great a work. But ſince his time, Mr. JOHN MACHIN, ſecretary to the Royal Society carried it to an hundred places in decimals, and demonſtrated, that if the circle's diameter be 1, the circumference will be 3.141592653,5897932384,626438327,9502884197,1693993751,0582097494,4592307816,4062862089,9862803482,5342117067,9. This amazing accuracy has rendered all attempts ſince ſuperfluous.

But, as a very ſmall degree of accuracy is ſufficient in common practice, the proportion of ARCHIMEDES, viz. as 7 to 22; or, that of METIUS, as 113 to 355; is commonly uſed; though that of VAN CULLEN, viz. as 1, to 3.1416 ſeems better adapted to Menſuration, as it ſaves the trouble of diviſion.

## PROB. I.

*To find the circumference of a Circle, the diameter being given.*

By the assistance of either of the above proportions, the circumference of the circle is easily found, when the diameter is given.

### EXAMPLE.

Let it be required to find the circumference of a circle, whose diameter is 120 feet, yards, &c.

1 By ARCHIMEDES' proportion.

```
As 7 : 22 : : 120
               22
              ---
              240
             240
             ----
        7) 2640 (377.1428, &c. The circumference required.
            21
            --
             54
             49
             --
             50
             49
             --
              10
               7
              --
               30
               28
               --
                20
                14
                --
                 60
                 56
                 --
                  4
```

2. By

2. By METIUS' proportion.

```
As 113 : 355 : : 120
              120
            -----
             7100
            355
            -----
113) 42600 (376.9911, &c. The circumference.
     339
     -----
      870
      791
     -----
       790
       678
     -----
       1120
       1017
     -----
       1030
       1017
     -----
        130
        113
     -----
         170
         113
     -----
          57
```

3. By VAN CULLEN's proportion.

```
As 1 : 3.1416 : : 120
                  120
            ---------
              628320
             31416
            ---------
            376.9920 The circumference required.
```

By either of these methods, my dear Discipulus, the circumference of the circle may be found sufficiently near the truth, for common practice: I say sufficiently near the truth, because it is impossible to solve the Problem absolutely accurate.

*Discipulus.* I perceive, Sir, that the two last are so nearly alike, that either of them may be used, as they give nearly the same result; but there is a considerable difference between them and the first; and therefore, I should reject it, unless it be the most accurate.

*Philo.* It has no claim to preference on that account: either of the latter giving the anſwer nearer the truth. It is, indeed, much eaſier than the ſecond; but the third is preferable, both on account of its eaſe and accuracy.

## PROB. II.

To find the area of the circle, the diameter and circumference being given.

The area of any circle is equal to the area of a Triangle, whoſe baſe is equal to the circumference, and its perpendicular equal to the Radius, or half the diameter. And hence the following

### RULE.

Multiply half the circumference, by half the diameter; the product will be the area required.

### EXAMPLE.

Suppoſe the diameter be 120, and the circumference 376.992, what is its area, or ſuperficial content?

### OPERATION.

188.496 Half the circumference.
60 Half the diameter.
11309.760 The area required.

The area of the circle may alſo be found, by multiplying the ſquare of the diameter, by 0.7854, the one fourth of the Periphery, when the diameter is 1.

### EXAMPLE.

Required the area of a circle, whoſe diameter is 120.

OPERATION.

OPERATION.

14400 Square of the diameter.
.7854 Common multiplier.

57600
72000
115200
100800

11309.7600 The area required.

PROB. III.

The diameter of a circle being given, to find the ſide of a ſquare, whoſe area ſhall be equal to the area of the circle.

All circles are to each other, as the ſquares of their diameters. But when the diameter of a circle is 1, its area is .7854. And hence we have the following

RULE.

As 1, the ſquare of 1, is to .7854, ſo is the ſquare of the given diameter, to the area of the given circle, the ſquare root of which, is the ſide of the ſquare required.

EXAMPLE.

Suppoſe the diameter of the given circle be 12, what is the ſide of the equal ſquare?

OPERATION.

As 1. to .7854, ſo is 144, the ſquare of 12, to the area.

144

31416
31416
7854

113.0976 The area of the circle.

The ſquare root of which is the ſide of the ſquare required.

```
          . . . .
Thus     113.0976 (10.634. The side of the square
          1                  required.
 206|    1309
   6|    1236
2123|     7376
   3|     6369
21264|    100700
    4|     85056
           15644
```

Or, if you multiply the diameter by 0.8862, half the square root of the circumference, when the diameter is 1, the product will be the area required.

```
Thus 0.8862
          12 The diameter.
     10.6344 The side of the square as before.
```

## PROB. IV.

The side of a square being given, to find the diameter of a circle, whose area is equal to the area of the square.

This Problem is the converse of the former; and therefore, if the side of the given square be multiplied by 1.12837, the product will be the diameter required.

### EXAMPLE.

What is the diameter of a circle, when the side of the equal square is 10.6344.

OPERATION.

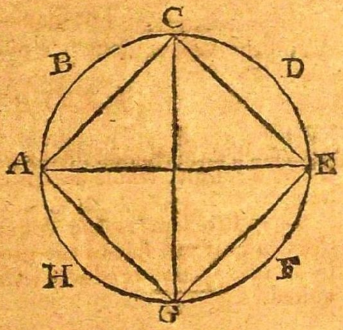
OPERATION.

```
    10.6344 Side of the ſquare.
     1.1283
    --------
     319032
    850752
   212688
  106344
 106344
-----------
 11.99879152 The diameter required, nearly
             equal to 12.
```

PROB. V.

The circumference of a circle being given, to find the ſide of the greateſt ſquare that can be inſcribed in that circle.

If the circumference of a circle be 1, the ſide of the greateſt ſquare, that can be inſcribed within it, is .2251: That is, if the circumference of the circle ABCDEFGH, be 1, the ſide AC = CE = EG = GA, of the greateſt inſcribed ſquare AC EG, will be .2251. Hence the following.




RULE.

Multiply .2251, by the circumference, and the product is the ſide of the ſquare required.

EXAMPLE.

Suppoſe the circumference of the circle be 26, what is the ſide of the greateſt ſquare that can be inſcribed in it?

### OPERATION.

```
  .2251
     26
 ------
  13506
  4502
 ------
 5.8526 The side of the square required.
```

## PROB. VI.

The diameter of a circle being given, to find the side of the greatest square that can be inscribed in that circle.

If the diameter of a circle be 1, the side of the greatest inscribed square will be .7071: And hence this general

### RULE.

Multiply .7071, by the diameter, and the product will be the side required.

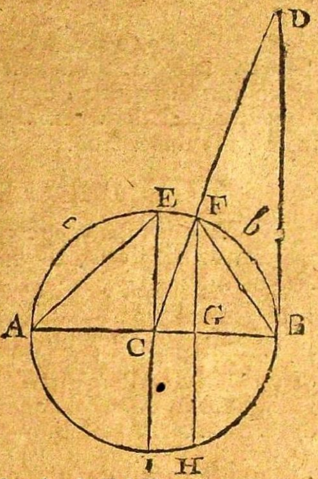
### EXAMPLE.

What is the side of the greatest square that can be inscribed in a circle whose diameter is 140?

### OPERATION.

```
  .7071
    140
 ------
 282840
 7071
 ------
 98.9940 The side of the square required.
```

Before we proceed farther in solving the Problems, with regard to the circle, I shall here, my dear Discipulus, explain several terms necessary to be understood, before you can proceed with ease and pleasure. For it is my intention to render every thing familiar, and imprint the principles on which each rule is founded, firmly in your memory.




In the annexed scheme of the circle AEFBHI,

AB is the diameter.

AC=CB=CE=CI, the Radius or semi-diameter.

AEC=AIC=BCE=BCI, a quadrant, or a quarter of the circle.

FCB, a Sector of the circle.

FB, the Chord of the arch F *b* B.

C D, the Secant of the arch F *b* B.

BD, the Tangent of the arch F *b* B.

FG, the Sine of the arch F *b* B.

GB, the versed Sine of the Arch F *b* B.

FGH, the Chord of the arch F BH.

AE, the chord of the arch A *c* E.

These definitions, my dear Discipulus, I beg you will imprint carefully in your memory; as I shall often mention them when I come to measuring heights and distances.

*Discipulus.* I should very ill deserve the trouble you have taken on my account, if I should neglect any thing you desire; and therefore give the closest attention to your instructions in every particular.

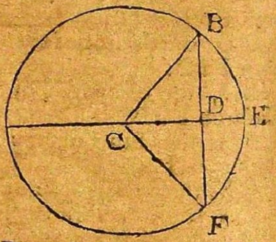
*Philo.* I well know that you give the utmost attention to every precept; but I thought it necessary to give you this caution, because it will save a great deal of trouble, when it is absolutely necessary to use these terms. But there is something more than a bare explanation of terms to be learned from this scheme. The nature and properties of the lines themselves should be considered: thus for instance,

The Radius, the Tangent, and the Secant, form a right-angled triangle, and if the Radius of the circle and the arch be given, both the Secant and Tangent may be easily found.

The Sine of an arch, its Chord and verſed Sine form another triangle, whoſe ſides increaſe in proportion as the arch increaſes. There are ſeveral other particulars, which a little attention of your own mind will readily ſuggeſt, eſpecially when the following Problems are thoroughly underſtood.

## PROB. VII.

The Radius of any circle, and the Chord of an arch thereof, being given; to find the verſed Sine of that arch.




In the annexed ſcheme CE=CB, the Radius of the circle, and BF, the Chord of the arch BEF, are given; to find DE, the verſed Sine of the arch BE=½ BEF.

But CB the Radius, BD =½ BF, the given Chord, and CD, part of the Radius, form a right-angled triangle, in which the Hypothenuſe CB, and the perpendicular BD, are given to find CD; which may eaſily be done by the 47th propoſition of EUCLID's firſt book, where it is demonſtrated, that the ſquare of the Hypothenuſe of any right-angled triangle, is equal to the ſquare of the perpendicular, added to the ſquare of the baſe: and, conſequently, if the ſquare of the perpendicular be ſubtracted from the ſquare of the Hypothenuſe, the remainder will be the ſquare of the baſe; the ſquare root of which being extracted, will give the baſe required; and the baſe CD, being ſubtracted from the Radius CE, will give DE, the verſed Sine of BE, or half the arch BEF.

### EXAMPLE.

Suppoſe the Radius CB = CE, be 170, and the Chord BF 180, required the verſed Sine DE.

### OPERATION.

170×170=28900 Square of the hypothenuſe CB:
And 90=BD=Half the chord BF.

```
 90
 90
8100 Square of the perpendicular BD.
```

Then 28900—8100=20800 the ſquare of the baſe CD.

```
        20800 (144,2
        1
 24|    108
         96
 284|   1200
        1136
 2882|  6400
        5764
         636
```

And the Radius 170—144.2=25.8 the verſed Sine DE required.

## PROB. VIII.

The Radius of a circle, and the verſed Sine of an arch being given, to find the Chord of twice that arch.

This is only another part of the above problem, the Radius, or Hypothenuſe, and baſe of the ſame right-angled triangle are given, to find the perpendicular, the double of which is the Chord required.

### RULE.

From the Radius take the verſed Sine, the remainder will be the baſe of the triangle: Then take the ſquare of the baſe, from the ſquare of the Hypothenuſe, and the remainder with the ſquare of the perpendicular, the ſquare root of which being extracted, and multiplied by 2, will be the Chord required.

EXAMPLE.

## EXAMPLE.

Suppose the Radius be 170, and the versed Sine 26.2. required the Chord of the double arch.

## OPERATION.

170—26.2=143.8, The base.

```
   170
   170
 -----
 11900
 170
 -----
 28900 Square of the Hypothenuse.

   143.8
   143.8
 -------
   11504
   4314
  5752
 1438
 --------
 20678.44 Square of the base.
```

Then 28900—20678.44=8221.56 The square of the perpendicular.

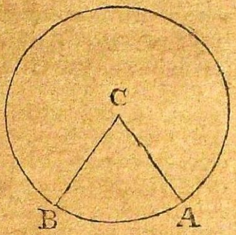
```
         8221.56 (90.67 The perpendicular, which
         81               being doubled, is 181.34,
         -------          the Chord required.
 1806|    12156
         10836
         -------
 18121|  132000
         126889
         -------
           5111
```

## PROB. IX.

To find the area of a circular Sector, the Radius and length of the sectoral arch being given.

A Sector

A Sector of a circle, is a figure comprehended under two Radii, or semi-diameters, and part of the circular arch: thus in the figure annexed, CBA is a Sector, comprehended under the two Radii, CB, and CA, and the arch, BA, and the area of it is found by the following




RULE.

Multiply the Radius by half the length of the circular arch, and the product is the area or superficial content required.

EXAMPLE.

Suppose the Radius CB=CA, be 20 feet, and the length of the arch 24 feet; what is the area of the Sector CBA.

OPERATION.

20 The Radius.
12 Half the arch.
240 The area required.

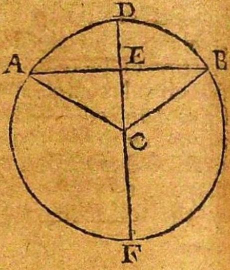
This Rule is founded upon that already given for finding the area of a circle, when the diameter, and consequently the circumference, are given; viz. to multiply half the circumference, by the Radius or half the diameter.

PROB. X.

To find the area or superficial content of any Segment of a circle, when the height and length of the chord of its arch are given.

A Seg-

A Segment of a circle, is a figure terminated by a right line, or chord, less than the diameter of the circle, and by a part of the circumference. Thus in the annexed figure, ABD is a segment of that circle, contained under the Chord of AB, and the circumference ADB and its area may be found by the following




## RULE.

Multiply the height of the Segment by 0.626; add the square of the product to the square of half the Chord: multiply twice the square root of the above sum by two thirds of the height, and the product will be the area required.

## EXAMPLE.

Suppose ED, the height of the circular Segment, be 11 feet, and AB, the Chord of the arch, be 37, what is the area of the Segment ABD.

## OPERATION.

```
   18.5
   18.5
 ------
    925
  1480
  185
 ------
 342.25 Square of half the Chord.
```

0,626

```
      0.626
         11  Height of the Segment
  ---------
      6.886
      6.886
  ---------
      41316
     55088
    55088
   41316
  ---------
   47.416996
   342.25
  ---------
   389.666996 (19.988.
   .  .  .  .

       1
      ------
  29| 289
      261
      ------
 299| ·2866
       2601
      ------
2988| ·26569
        23904
      --------
29968| ·266596
         239744
        -------
        ·26852

      19.988
        7.32  Two thirds of the height.
     ---------
      39976
     99964
    139916
    ----------
    146.31216  The area required.
```

This Problem may alſo be ſolved in a very different manner, viz. by finding the ſemi-diameter of the circle DC = to CB = CA. This being done, the area of the Sector CABD will be very eaſily found; and alſo the area of the triangle ABC: Then if the area of the triangle ABC be ſubtracted out of the area of the Sector, the remainder will be the area of the Segment ADB.

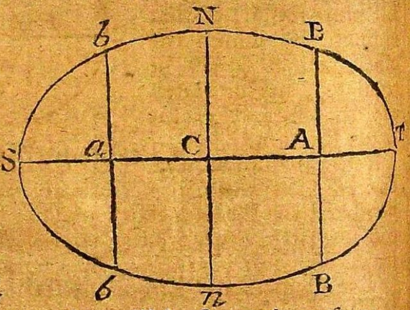
By the thirtieth propoſition of EUCLID's ſixth book, half the Chord of any arch is a mean proportional, between

tween the verſed Sine, and the remaining part of the diameter. If therefore you ſquare AE, or EB, half the Chord AB, and divide that ſquare by the verſed Sine ED, the quotient will be EF, or the part of the diameter wanting; this added to the verſed Sine ED, will give the whole diameter DF, the half of which will be the ſemi-diameter CD=CA=CB required. But the former rule ſeems preferable; tho' both will give the ſame reſult.

## SECT. IX.

### *Of Meaſuring an* ELLIPSIS.

An Ellipſis, vulgarly called an Oval, is a figure bounded by a regular curve line, returning into itſelf, and formed not from one, but from two centers, called the Foci of the Ellipſis. Its two diameters are unequal, the longer of which is called the tranſverſe diameter, and the ſhorter the conjugate diameter.




Thus the adjacent figure SNTBC is an Ellipſis; ST, the tranſverſe diameter: N*n*, the conjugate diameter; *bb*, BB, two ordinates; *a*, A, the two Foci of the Ellipſis: and by their aſſiſtance, the Periphery may be drawn in a manner that will be ſhewn hereafter.

If the tranſverſe diameter ST be interſected, or divided into equal parts, by an ordinate rightly applied, that is, drawn parallel to the conjugate diameter, as at the the points *a*, C, A, &c: Then are thoſe parts TA, TC, T*a*, and S*a*, SC, SA, uſually called Abſciſſæ, or parts cut off; and the Rectangle of any two parts, being added together, will be equal to the tranſverſe diameter: that is,

TA

$TA + SA = TS$, and $TC + SC =$ to $TS$:
Or, $Ta + Sa = TS$, &c.

Every Ellipſis is proportioned, and all ſuch lines as relate to it are regulated, by the following

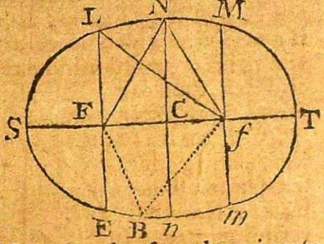
THEOREM.

As the Rectangle of any two Abſciſſæ in general: is to the ſquare of half the ordinate which divides them : : ſo is the Rectangle of any two particular Abſciſſæ: to the ſquare of half that ordinate which divides them.

There are ſeveral methods uſed by authors, for finding the Latus Rectum, or third principal line in the Ellipſis, but I think none ſo eaſy as the following

THEOREM.

As the tranſverſe diameter : is to the conjugate : : ſo is the conjugate : to the Latus Rectum :




That is, as $TS : Nn$ : : $Nn$ : to $LE$, the Latus Rectum.

Hence it is evident that LE thus found, is that ordinate by which the other ordinates may be regulated, and diſcovered: therefore it is the Latus Rectum.

Hence it follows, that if the tranſverſe and conjugate diameters of any Ellipſis are given either in lines or numbers, the Latus Rectum may be eaſily found; and, conſequently, any ordinate, whoſe diſtance from the conjugate diameter is given.

The focus of any Ellipſis, is the diſtance of the Latus Rectum from the conjugate diameter, or middle of the Ellipſis; and that diſtance is always a proportional between the half ſum, and the half difference of the tranſverſe and conjugate diameters. And hence this general

THEOREM.

From the ſquare of half the tranſverſe diameter, ſubtract the ſquare of half the conjugate; the ſquare root of

of their difference, will be the distance of each focus from the middle or common center of the Ellipsis.

That is, supposing the points $f$ and F to be the true Foci, viz. $FC = Cf$, and $TC = \frac{1}{2} TS$, $NC = \frac{1}{2} Nn$. Then as

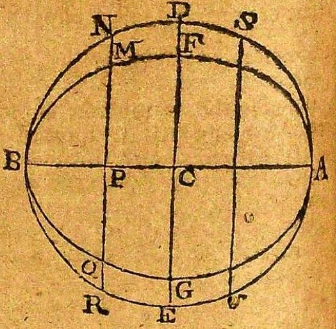
$TC + NC : fC :: FC : TC - NC$. Therefore the square of FC = the square of TC — the square of NC. Consequently $FC = \sqrt{TC^2 - NC^2}$. And hence is derived that noble proposition on which both the usual method of describing an Ellipsis, and drawing Tangents is founded.

## Proposition.

If from the two Foci of an Ellipsis, there be drawn two right lines, so as to meet each other in any point of the Periphery of the Ellipsis, the sum of these lines will be equal to the transverse diameter: that is

$fN + FN = TS$. $fL + FL = TS$. Or, $FB + Bf = TS$.

As this proposition holds good for every point in the circumference of the Ellipsis, it is evident, that if the ends of a thread just the length of the transverse diameter TS, be put round two pins, fixed in the Foci of the Ellipsis, and moved round by a point in the extremity, that point will describe the true Periphery of the Ellipsis.




Let AB, be the transverse diameter of an Ellipsis AFBG; GF its conjugate diameter, and ADBE, its circumscribing circle: then,

Because in the circle as $AC \times CB : AP \times BP ::$ the square of CD : the square of PN; and in the Ellipsis $AC \times CB : AP \times BP ::$ the square of CF : the square of PM; therefore it will be as, CD : CF : PN : PM; or as AB : FG : NR: to MQ. But all the NR's, SS's compose the area of the

the circle, and the ſame number of MQ's form that of the Ellipſis. Hence as AB : FG, or the longer diameter of the Ellipſis to the ſhorter, ſo is the area of the circumſcribing circle, to the area of the inſcribed Ellipſis. Wherefore as the ſquare of the diameter of the circumſcribing circle, is to the Rectangle of the tranſverſe and conjugate diameters of the Ellipſis; ſo is the area of the circle, to the area of the Ellipſis.

Hence as the circle is to its circumſcribed ſquare, or to the ſquare of its diameter; ſo is the Ellipſis to its circumſcribed Parallelogram, or to the Rectangle of its two diameters.

Hence the area of an Ellipſis may be eaſily found: for the ſquare root of the Rectangle of its two diameters, will give the diameter of a circle, whoſe area is equal to the area of the Ellipſis.

Hence, alſo, as the area of any Segment of the Ellipſis, is to the area of the correſponding circular Segment, ſo is the conjugate diameter of the Ellipſis, to its tranſverſe.

Hence the area of any Ellipſis is found by theſe general rules.

### RULE I.

Multiply the tranſverſe diameter, by the conjugate, and that product by .7854, the area of a circle, whoſe diameter is 1; the laſt product will be the area required.

### EXAMPLE.

Suppoſe the tranſverſe, or longeſt diameter be 40.5, and the conjugate or ſhorteſt 30.4; required the area of the Ellipſis.

OPERA-

### OPERATION.

```
       40.5
       30.4
     ------
       1620
     12150
   --------
    1231.20
      .7854
   --------
     492480
    615600
   984960
  861840
  ----------
  966.984180
```
The area required.

### RULE II.

Extract the square root, out of the Rectangle of the transverse and conjugate diameters, and the result will be the diameter of a circle, whose area is equal to the area of the Ellipsis.

### EXAMPLE.

Suppose the transverse diameter be 40.5. and the conjugate 30.4; what is the area?

### OPERATION.

```
     40.5
     30.4
   ------
     1620
   12150
```

```
       . . .
     1231.20 (35.088 The diameter required.
       9
65|  331
     325
7008| 62000
      56064
70168| 593600
       561344
        32256
```

But the circumference of a circle, whose diameter is 1, is 3.1416; consequently the circumference of the given circle is 3.1416×35.088, or 110.232, &c. the half of which is 55.116: consequently.

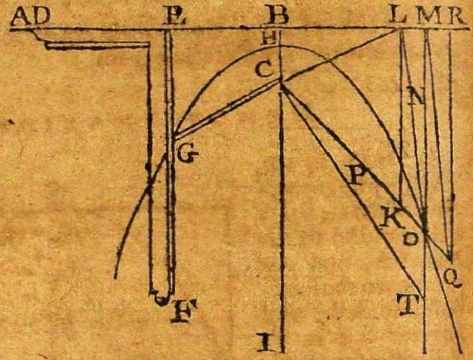
```
   55.116 Half the circumference.
   17.544 Half the diameter.
   ------
   220464
  220464
 275580
385792
55116
----------
966.935104 The area required, nearly
              the same as before.
```

## SECT. X.

### *Of Measuring a* PARABOLA.

A Parabola is a curvilineal figure, made by the section of a Cone, cut by a Plane parallel to by one of its sides.

In order to explain this definition, which will be clearly understood, when I come to shew the manner of finding the solidity of a Cone, I will shew a method of describing it in Plano.



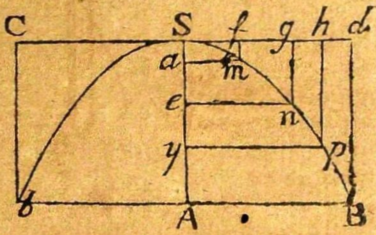

Draw a right line as IB, and assume a point C, without it; then in the same Plane, with this line and point, place a square rule DEF, so that the side DE may be applied to the right line AB, and the other FE turned to the side on which the point C, is situated. This done, and the thread FGC, exactly the length of the side of the rule FE, being fixed at one end of the extremity of the square, as at F, and at the other to the point C, if you slide the side of the rule DE along the right line AL, and by means of a pin, G, continually apply the thread to the side of the rule EF, so as to keep it always stretched as the rule is moved along, the point of this pin will describe the Parabola GHO.

In the above figure, the right line AL is called the Directrix. The point C is the Focus of the Parabola. All perpendiculars to the Directrix, as LK, RQ, are called diameters; the points where these cut the Parabola, are called its Vertices; the diameter BI, which passes through the Focus C, is called the Axis of the Parabola, and its Vertex, H, the principal Vertex. A right line terminated on each side by the Parabola, and bisected by a diameter, is called an Ordinate Applicate, or simply the ordinate to that diameter. A line, equal to four times the Segment of that diameter, intersected between the Directrix, and the Vertex is called the Latus Rectum, or Parameter of that diameter. A right line which touches the Parabola only in one point, and being produced on each side falls without it, is a Tangent to it in that point.

The

The area of every Parabola is equal to two thirds of its circumſcribing Parallelogram.

## DEMONSTRATION.




Let the figure, SAB, repreſent half the Parabola; draw *d*B, parallel to the Axis SA, and S*d* parallel to the ſemi-ordinate AB; and ſuppoſe S*d* to be divided into an infinite ſeries of equidiſtant points, as *f*, *g*, *h*, &c. and from theſe points imagine a ſeries of parallel lines, viz. *fm*, *gn*, *hp*, &c. to touch the curve of the Parabola, and meet the ſemi-ordinates, *ma*, *ne*, *yp*.

Then it will be $\begin{cases} \text{As } SA : AB^2 :: Sa : am^2. \\ \text{As } SA : AB^2 :: Se : en^2. \\ \text{As } SA : AB^2 :: Sy : yp^2, \text{ \&c.} \end{cases}$

But $Sa = fm$. $Se = gn$. $Sy = hp$. $SA = dB$.

Therefore alternately it will be $\begin{cases} AB^2 : dB :: yp^2 : hp. \\ AB^2 . dB :: en^2 : gn. \\ AB^2 : dB :: am^2 : fm. \end{cases}$

In theſe proportions $am^2$, $en^2$, $yp^2$, &c. are a ſeries of ſquares, whoſe roots S*f*, S*g*, S*h*, &c. are in arithmetical progreſſion, beginning at the point S. And becauſe the lines *hp*, *gn*, *fm*, &c. have the ſame Ratio; therefore they are a ſeries of ſquares, wherein *d*B, is the greateſt term, and S*d*, the Number of terms. Conſequently $dB \times Sd =$ the ſum of thoſe lines. But $SA \times \dot{A}B = \overline{dB \times Sd}$. : Therefore $\overline{SA \times \overset{3}{AB}} =$ the ſum of allthe ſeries of lines; but all thoſe lines conſtitute the area of the ſemi-parabola's complement; or the area of what half the Parabola wants of completing, or filling up the Parallelogram S*d* AB.

Wherefore $\overline{SA\times AB}-\frac{1}{3}\overline{SA\times AB}=\frac{\overline{2SA\times AB}}{3}$, will be the area of half the Parabola SAB. Consequently $\frac{2}{3}SA\times bB$, will be the area of the whole Parabola $bSB$.

## EXAMPLE.

Suppose $bB$ the base, or greatest ordinate of a Parabola, 24; and SA its intercepted diameter, or Axis 33; required the area, or superficial content?

## OPERATION.

$$
\begin{array}{r}
66=2\,SA. \\
24=b\,B. \\
\hline
264 \\
132\phantom{0} \\
\hline
1584=2SA\times bB.
\end{array}
$$

Then 3) 1584 (528. The area required.

$$
\begin{array}{r}
15\phantom{84} \\
\hline
8\phantom{4} \\
6\phantom{4} \\
\hline
24 \\
24 \\
\hline
\end{array}
$$

# BOOK II.

*The application of the above Rules to the actual* MENSURATION *of all kinds of* SUPERFICIES.

*Philo.* WE have now, my dear Discipulus, gone through the several rules, on which the art of practical Mensuration is founded; let us therefore now proceed to shew the methods used by artists in putting those rules in practice: and I will venture to say, that if you listen with the same attention you have hitherto done, you will find no difficulty in this part of the task.

*Discipulus.* You may depend upon it, Sir: nothing in my power shall be wanting. Indeed it would be very ungrateful, if I did not set the highest value on your instructions, as I have already profited by them in so remarkable a manner.

*Philo.* It gives me the highest pleasure to find you are so fond of your learning: I wish all youths would imitate your example; they would never then think their tasks irksome, nor the pains they took in understanding their lessons spent in vain. But leaving these reflections, we will proceed to shew the best methods of putting in practice the foregoing rules of Superficial Mensuration.

## CHAP. I.

*Of the art of* MEASURING LAND; *generally called* SURVEYING *of* LAND.

SURVEYING, or the Art of Measuring Land, implies the act of taking the dimensions of any field, parcel, or tract of land, laying down the same in a map or draught, and casting up the contents, by the methods already delivered for the mensuration of superficies.

As this art is of the utmost importance to all owners and occupiers of land, I shall endeavour to explain it in the easiest and most intelligible manner; and shew how any field, of whatever figure it be, may be measured, protracted, and cast up, with the help of chains, compasses and scales: and afterwards endeavour, in the

moſt eaſy and conciſe manner, to ſhew the uſe of the plain table and other inſtruments uſed in ſurveying.

## SECT. I.

### *Of Surveying by the* CHAIN, COMPASSES, and SCALES *only.*

This method of meaſuring land is generally uſed where the quantity is ſmall, as that of one, two or three fields; and therefore often called for: as the farmer frequently agrees with reapers and mowers, to reap or mow his corn at a certain price, for each acre. But before we proceed to explain this method of ſurveying, it will be proper to deſcribe the neceſſary inſtruments.

### *Concerning* CHAINS, COMPASSES, *and* SCALES.

I. Amongſt the many ſorts of chains uſed for meaſuring land, three are chiefly eſteemed, and which bear the names of their inventors, Rathbone, Gunter, and Wing; all of them ingeniouſly divided, and uſeful in their kind; but the deſcription of one being ſufficient for our purpoſe, we ſhall deſcribe that invented by Mr. Gunter, as being moſt in uſe, and eaſy to be procured.

This chain contains in length four ſtatute-poles or perches, each perch containing 16 feet and a half, or 5 yards and a half; ſo that the whole chain is 66 feet, or 22 yards long.

This chain is divided into 100 equal parts, or links, whereof 25 are a juſt pole or perch; and for ready counting, there is uſually a remarkable diſtinction by ſome plate or large ring at the end of every 25 links, but eſpecially at the middle of the chain, which ſhould differ from the reſt in greatneſs and conſpicuouſneſs. Alſo at the end of every tenth link, it is uſual to hang a ſmall curtain-ring; and if there be at every five links end a piece of wire, made like the bow of a link, with a ſmall ſhank, an inch or leſs long, or ſome ſuch diſtinction, it will be ſtill better.

When you are to meaſure any line by this chain, you need regard no other denomination than chains and links, ſet down with a dot of your pen betwixt them.

For example, if you find the ſide of a cloſe to be 6 chains and 35 links long, it is thus to be put down, 6. 35.

But if the links be under 10, a cypher muſt be prefixed; ſo 7 chains 9 links muſt be thus expreſſed 7. 09.

II. In the uſing of this, or indeed any other chain, care muſt be taken both to go ſtrait, and to keep a true account; for which purpoſe it is neceſſary that the perſon who goes before ſhould carry a bundle of rods, to ſtick down at one end of the chain, having firſt ſtretched it well; and that he who follows, ſhould gather up the rods, keep the account exact, and alſo, at every remove, obſerve whether the leader is directly between his eye and the angle, or any other mark, towards which he is meaſuring.

III. Compaſſes are ſo well known, that it is needleſs to give any other deſcription of them, except that they ſhould be made of braſs, with ſteel points, ſmall and neatly wrought, 9 or 10 inches long, from the joint to the points, turning ſo truly upon the rivet, that they may be eaſily opened, and yet ſtand ſo firmly, that an arch or circle may be, without their ſhrinking, deſcribed upon a large Radius.

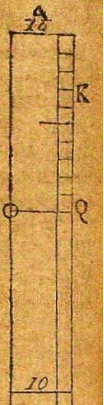
IV. Scales are certain lines divided into equal parts, upon plates or broad rules of braſs or box, and they are of two ſorts, *firſt*, plain; *ſecond*, diagonal.

1. Plain ſcales are made up of two ſmall lines, parallel to one another, and at a little diſtance, and theſe are divided into large equal parts, which ſignify tens, and are noted 10, 20, 30, 40, 50, &c. according to the length of the lines.

They may be of any convenient length; but theſe large diviſions are ſeldom more than inches, or leſs than third parts of an inch.

Again one of the large diviſions is ſub divided into ten equal parts by ſhort lines, whereof that in the middle, ſtanding for five, is longer than the reſt.

According to the numbers of theſe little parts contained in an inch, it is named, A Scale of 10, 11, 12, 16, 20, 24, 30, &c. in an inch. The annexed figure marked A, is of 10 in an inch, ſo noted at the top, as is uſual upon the rules, and indices of plain tables. The line marked O Q, ſeparates units and tens; units being taken upward from that line, and tens downward, but mixt numbers both ways.



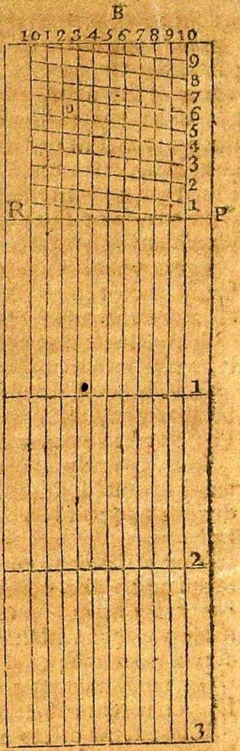

EXAMPLE.

Seven is the extent of the compaſſes upon the ſcale A, from the line OQ to K; 30 is their extent from the line OQ to the bottom; and 27 is the extent from the line 20, to the ſhort line K.

Theſe plain ſcales, eſpecially the ſmaller ſorts of them, ſuch as 24, or 30 in an inch, are very proper for drawing figures upon paper, where the numbers repreſented by the lines are not above 100, for then every diviſion may be counted as it is upon the ſcale, or above upon a longer ſcale.

Alſo, in the ſurveying of foreſts, chaſes, and extenſive commons, where the lines are vaſtly long, and the miſtake of a few lines is not conſiderable, they may be conveniently uſed, accounting the tens and units, for ſo many whole chains, and ſo eſtimating the parts of a chain with the compaſſes, upon the ſmall diviſions. But it would be much better for ordinary meaſuring, if the grand diviſions of the ſcale were each two inches, for then the ſmaller diviſions being of 5 in an inch, would be ſo large as to be ſubdivided into 5 each, which repreſents 20 links; and then the half of one of thoſe leſſer diviſions, ſignifying 10 links, and the quarter 5, it is rendered eaſy for the moſt common capacity, to come very near the truth by eſtimation.

2. The




2. The Diagonal Scale is ſo well known to every mathematical inſtrument-maker, ſo eaſy to be procured, and every way ſo fitted to Gunter's chain, that we cannot but highly recommend it to every ſurveyor.

It is made as appears by the figure in the margin, upon 11 parallel equi-diſtant lines, ſo as to include 10 equal ſpaces, which are all cut at right angles by tranſverſe lines, dividing them all into 4 equal parts.

One of theſe tranſverſe lines, viz. PR, where it toucheth the firſt and laſt lines, ſeparates between the hundreds, or whole chains, and the tens, repreſenting 10 links a piece, the chains being numbered downwards on the left hand from P only to 3, but on the inſtrument itſelf they may go on to 9 or 10; but the tens upwards from 1 to 10.

From the points of diviſion into tens upon the firſt line, beginning at P, to the like points beginning at R in the laſt line, are 9 Diagonal lines drawn, the firſt beginning at P, and ending at the firſt diviſion above R. The ſecond beginning at the firſt diviſion above P, and ending at the ſecond above R. In a word, they are all drawn from one diviſion leſs from P, to one more from R; by which means every Diagonal, by that time it hath paſſed from the firſt line to the eleventh, is a whole tenth part of an inch, which anſwers to ten links of the chain, farther diſtant from the

the line P R, than at the points upon the firſt line whence it was drawn.

Every one of theſe Diagonals is divided into ten equal parts, by the long parallel lines running through the whole of the ſcale, and numbered on the top from 1 to 9. Whereby it is evident, that the interſection of the 9 parallel lines that are numbered at the head with any Diagonal, muſt be farther diſtant from the line PR, than the interſection of the line next before it, with the ſame Diagonal by $\frac{1}{10}$ of $\frac{1}{10}$; that is, by $\frac{1}{100}$, which anſwers to a ſingle link of your chain.

From what hath been ſaid, and the inſpection of the figure, theſe things plainly follow: which, as ſo many clear inſtances, will help you to underſtand it fully.

The diſtance from P R, to the ſecond diviſion below it, anſwereth to two chains.

The diſtance from PR, to the eighth diviſion upward, being taken, with compaſſes, upon the firſt line of the 11 from P to 8, anſwereth to 80 links.

Conſequently the extent of the compaſſes from the ſecond grand diviſion below P, to the eighth of the leſs diviſions upward, is proportionable to two chains 80 links.

The diſtance from PR, to the firſt Diagonal being taken upon the parallel line, noted with 9 above, anſwers to 9 links: Where obſerve, that the firſt Diagonal is not that which is noted with 1, but that which is drawn from the point P.

The diſtance upon the ſame line, from P R to the Diagonal that is marked with 7, is equal to 79 links.

The extent of the compaſſes from the bottom of the figure upon the ſame line to the ſame Diagonal anſwers to 3 chains 79 links.

In ſhort, the chains may be meaſured upon any line PR, to the given diviſion noted with the given number Decads alone, or chains and Decads upon the firſt line where the Diagonals begin. Links alone, Decads with links and chains, and Decads with links always upon that line, where the number of the odd links ſtand at the head of the ſcale.

How

*How to cast up the content of a Figure, the lines being given in chains and links.*

Having described these plain instruments, and in some measure shewed the use of them in general, it is very proper in the next place, to teach their joint use in measuring and protracting; but because I would have you and every young surveyor, before I take him into the field, able to perform the work entire, I intend to shew:

1. How he ought to make his computations. 2. The grounds, or principles that will justify him in so doing.

For the first, the following rules are necessary.

1. Put down your length and breadth of squares and oblongs, and your bases and half perpendiculars of triangles, directly under one another, expressed by chains and links, with a dot betwixt, as we have before observed.

2. If the odd links are under ten, put a cypher before the numeral figure expressing them; and if there be no odd links, but all even chains, put two cyphers after the dot.

3. Multiply length by breadth, and base by the half perpendicular, according to the rules already given for finding the content of figures.

4. From their product cut off five figures, accounting cyphers also for such, reckoned from the right hand backward, with a dash of your pen, so shall these to the left hand signify acres.

5. If those 5 figures cut off be not all cyphers, multiply them by 4, and cutting off 5 towards the right hand again, the rest will be roods, or quarters.

6. If amongst these 5 figures towards the right hand, that were cut off at the second multiplication, there be any figures besides cyphers, multiply all the 5 by forty, and cutting off 5 again by a dash of your pen, those on the left hand signify square perches, poles, or roods.

This will be more fully explained by the following examples.

## QUESTION I.

What is the content of a ſquare whoſe ſides are every one of them 7 chains, 25 links?

```
Length  7.  25.
Breadth 7.  25.
        -------
          3625
         1450
        5075
        -------
        5|25625
             4
        -------
        1|02500
            40
        -------
        1|00000
```

ANSW. Five acres, 1 Rood, and 1 Perch, as appears by the above operation.

## QUESTION II.

In a long ſquare whoſe length is 14 chains, and the breadth 6 chains 5 links, what is contained?

```
Length  14.00
Breadth  6.05
        -----
         7000
       8400
      -------
       8|47000
             4
      -------
       1|88000
           |40
      -------
      35|20008
```

ANSW. Eight acres, 1 rood, and 35 perches, as the Work makes it evident.

QUESTION III.

In a triangle, whose base is 3 chains, and half the perpendicular 98 links, what is the content?

| | |
|---|---|
| The base | 3.00 |
| Half perpend. | 0.98 |
| | 2400 |
| | 2700 |
| | \|29400 |
| | 4 |
| | 1\|17600 |
| | 40 |
| | 7\|04000 |

ANSW. No acres, 1 rood, 7 perches.

There are other ways of computation by scales, tables, &c. as will appear by the following observations.

1. It is evident, that by this method of multiplication, the product is square links; for every chain being 100 links, the multiplying 7.25 by 7.25, or 725 by 725 is the same with or without dots; for the dots signify something as to conception, but nothing in operation. The product therefore of the first example was really 525625 links.

2. Every chain being 4 perches long, it follows, that 5 chains, or 20 perches in length, and 2 chains, or 8 perches in breadth make an acre, or an 160 square perches; for 20 being multiplied by 8, gives 160.

3. From hence it plainly follows, that there are exactly 100000 square links in an acre; for 5 chains multiplied by 2, is the same with 500 links by 200, which makes 100000. And it is an old but plain rule, that when the divisor consists of 1 and cyphers, as 10, 100, 1000, 10000, 100000, &c. cut off from the right hand as many figures of the dividend as the divisor hath cyphers, accounting them the remainder; so will the rest on the left side be the quotient. It is evident then, that 525625 square links, make 5 acres, and 25625 square links over.

Thus, as far as relates to acres, we have made it plain to every capacity, that the rules for computation

are good. We ſhall next proceed to roods and perches, though it might be ſufficient to mention that known rule in decimal arithmetic, which multiplying decimal fractions by known parts, gives thoſe known parts in integers, due regard being had to the ſeparation. We ſhall proceed thus: if 25625 ſquare links, which remain above an acre, contain any quarter or quarters of an acre, then if they be multiplied by 4, and divided by 100000, that is 5 cut off from the product, they will contain ſo many acres as now they do quarters, or roods, for any number of quarters multiplied by 4, will of courſe produce the like number of units or integers; the diviſion only reduces them into the right denomination. Now, 25625 being multiplied by 4, and 5 figures being cut off from the product, the reſult is, 1|02500, that is an acre and above; which ſhews it was above a quarter before it was multiplied by 4.

And to find how much, that is, how many ſquare perches are contained in this laſt remainder, you muſt conſider this 2500, not as ſquare links remaining above the rood or quarter, but as 4 parts or quarters of ſquare links, or which is the ſame, as the true number of ſquare links, multiplied by 4, and conſequently being multiplied by 40, the fourth part of the ſquare perches in an acre, it muſt as often contain 100000 ſquare links, or an acre, as the quarter of this number 2500, *viz.* 625 ſignifying ſquare links, contains ſquare perches; for 100000 divided by 160, the number of perches in an acre, gives 6.25, anſwering to 1 perch, and 2500 multiplied by 40, gives 100000, or 1 acre, the 5 cyphers being cut off.

```
      2500        160      100  000625
        40                 960
   -------                 ----
   1,00000                 400
                           320
                           ----
                           800
                           800
                           ----
```

How

*How to measure a close, or parcel of land, and to protract it, and give up the content.*

All closes, or parcels of land, are either such as need not be plotted for finding their true measure, but the chain alone does the work; or such as cannot be conveniently measured without plotting or protraction.

Of the first sort are the square and long square, known before hand to be such, or found so to be by such instruments as are not yet described, or by measuring all the sides and Diagonals. These squares and Parallelograms need no protracting; for you need only multiply the chains and links of the breadth, and proceed as in the first and second examples; but all other figures whether triangular or multangular, are to be protracted. We shall give examples therefore in the 3 sorts of figures, Triangular, Quadrangular, and Multangular.

But before we proceed to particular instances, let the young practitioner remember,

1. To begin at some particular angle of the field, where there is a house, gate, stile, well, or other mark; and if there be none, then to dig up a clod, and drive a stake, or at least observe, whether it is situated in the east, west, north, or south corner of the field, and on your paper mark it with the letter A, or any other.

2. To go parallel to the side of the field, if you are not obstructed by pits, bushes, &c. and if you are, to allow for it; accustoming yourself to go constantly with the same hand towards the hedges, walls, or pales; and when you go contrary to your usual custom, note it on your paper, by some mark known to yourself.

3. To set down the chains and links of every side as you measure, and not trust your memory; for which purpose, a black lead pen will be very proper.

4. To take care if you have more scales than one upon your rule, that you do not confound yourself by taking off lines from several scales, or measuring perpendiculars upon wrong ones; for every line of the same figure must be made by the same scale, and the perpendiculars measured by it.



5. To

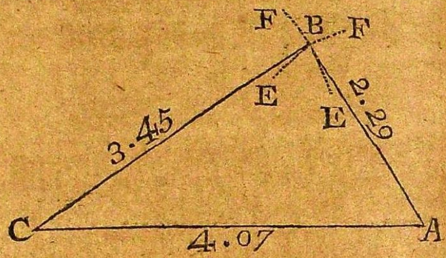
5. To make use of a scale of larger divisions when you measure small closes, and of smaller when you measure great ones.

6. To make your lines and points where angles meet, small, pure, and neat.

7. To set on your chains and links at twice, when any line is too long for your scale.

These things being premised, proceed thus:

I. Suppose a triangular field is to be measured with the chain: begin at the eastern angle A, and find the sides in their order and measure to be severally thus; 2.29, 3.45, 4.07.




Making use of the less Diagonal scale, because the other would make the figure too large, otherwise it were more proper for so small a close; with your compasses take off the scale 4 chains and 7 links, and setting them from A to C, draw that line for the base, as being the longest of the three; then take 2 chains 29 links off the same scale, and set them in the eastern point A, and turning the loose foot of the compasses above the line at C, describe at that distance 2 29, the arch E F.

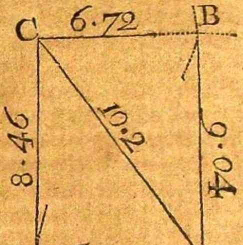
Next taking with your compasses upon the same scale, the extent of 3 chains 45 links, place one foot in the point C, and with the other make the arch F E, intersecting the former in the point B; and drawing the lines A B, and B C, the triangle A B C is the plot of the triangular field measured.

But before you can give up the content, you must find the length of the perpendicular, which is done by setting one foot of the compasses in B, and extending the other to the base AC, so as to touch, without passing over it, for then the length of the perpendicular is between the points of the compasses, and being applied

the points of the same scale, by which the triangle ABC was made, it appears to be 1 chain 42 links. With the half whereof multiply 4.07, the length of the base, the content appears to be no acres, 1 rood, 6 perches.

| | |
|---|---|
| The base | 4.07 |
| Half perpend. | 0.71 |
| | 407 |
| | 2849 |
| | 28897 |
| | 4 |
| | 1\|15588 |
| | 40 |
| | 6\|23520 |

II. Suppose you were to measure a quadrangular, or four-cornered field, begin, as before, at some remarkable angle, and going round the close find the sides to be 9.04, 6.72, 8.46, 7.28, and the Diagonal from that remarkable angle to the opposite angle to be 10.02; we begin to protract it thus:




Having by the help of your scale and compasses drawn the Diagonal 10.02, from the remarkable angle A to C, the opposite angle, make a triangle of it, and the first and second sides 9.04 and 6.72, and another after the same method of that Diagonal, and the third and fourth sides 8.46 and 7.28, so you have the Trapezium ABCD.

Then by the help of your scale and compasses, you will find the perpendicular of the triangle ABC to be 6.02, and of the other, viz. CDA 6.01, which added, are 12.03, whereof the half sum is 6.01; by which multiplying the base 10.02, the content of the field will be 6 acres, no roods, 3 perches.

| | |
|---|---|
| The baſe | 10.02 |
| The perpend. | 6.01 |
| | 1002 |
| | 60120 |
| | 6\|02202 |
| | 4 |
| | \|08808 |
| | 40 |
| | 3\|52320 |

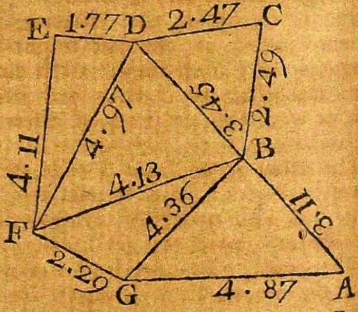
Before we proceed any farther, it is neceſſary to obſerve,

1. Any Quadrangular cloſe, or parcel of ground whatſoever, having right lines, may be thus meaſured, protracted, and computed.

2. The odd meaſure below perches is not regarded here, nor in the former computations, being always under a ſquare perch; but in Multangulars, where there are many remainders, they muſt be ſummed up, and the perches contained in them added to the content before found.

3. This, and the following figures, are made, that they might not be too large, by a ſcale of 400 in an inch, that is, by the leſſer Diagonal ſcale, each chain and link being counted two.

III. If the Multangular figure in the margin, be conceived to repreſent a cloſe of ſeven ſides, which is to be meaſured, begin at the remarkable angle A, and going round the cloſe, find the ſides, will be 3.11, 2.49, 2.47, 1.77, 4.11, 2.29, 4.87.




Then meaſure the four Diagonals, BD, DF, FB, and BG, in order, you will find them to be 3.45, 4.97,

4.97, 4.13, and 4.36; which is as ſhort a way as can be taken to prevent unneceſſary walks.

But when you come to protract by the help of your ſcale and compaſſes, firſt make the triangle BCD of the firſt diagonal, and the ſecond and third ſides. Then the triangle DEF upon the ſecond Diagonal, and fourth and fifth ſides; and upon the ſame Diagonal, as a common baſe, the triangle BDF of the firſt, ſecond and third Diagonals.

Next of the ſame third Diagonal, together with the fourth Diagonal and the ſixth ſide, make the triangle BFG, and upon the fourth Diagonal, as upon a common baſe with the firſt and laſt ſides, the triangle ABG. So is the whole cloſe plotted.

And now it ſtands viſibly reduced into two Trapezias ABFG and BDEF, together with the triangle BCD, which we ſhall not now caſt up, having already frequently ſhewn the method of doing it.

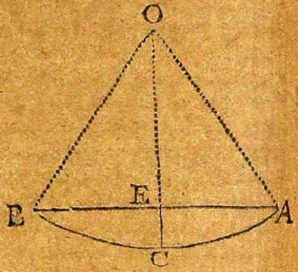
But it muſt be acknowledged that this method of plotting parcels of land, which have many angles, requires not only more care and pains, but better ſkill and memory, than to draw Diagonals upon paper, when the plot is already taken by the plain table, or other ſtanding inſtrument. We ſhall therefore to aſſiſt our young practitioner, inform him of two ways to prevent his being liable to any miſtake.

One is to divide the Multangular field into two or more parts, as the laſt might have been by the Diagonal BF; then might each part have been meaſured ſeverally, as if they had been ſeparated by a pale, or were the property of different men, parted by a boundary.

The other is to draw a rude draught of the figure of the land you intend to meaſure, not only as to the ſides, but alſo neceſſary Diagonals. Then meaſuring the lines upon the ground, correſpondent to thoſe on the paper, ſet the lines as you meaſure them upon the lines of the draught, as if it were the true one, and when you have finiſhed the meaſuring, protract it truly.

*Concerning the measuring of circles, and their parts.*

It is very rare, that a Land-Meter ever has occasion to measure any field, or parcel of land, that will prove either Circle, Semi-circle; Quadrant or Sector. Sometimes, indeed, there will be a little crook in an old hedge bowing like an arch; tho' I have never seen any offer to measure it as a Segment, but always take it as an angle, or angles.




But that which most frequently occurs in Mensuration, though seldom much regarded, except where a minute exactness is required, is that particular sort of Segment, which is less than a semicircle, such as this figure ABC. And to find the content of it, the center of the circle, whereof this is a Segment, must be first found, as here at O, from which lines drawn to A and B, make up the Sector AOBC; which being measured according to the last rule, and from the content thereof, the content of the triangle AOB subtracted, the difference or residue is the content of the Segment ABC.

But two questions may be here demanded.

1. How may the center be found? 2. How may such a portion of land be truly protracted and computed?

To the first we answer, That the most exact and artificial way is, by making a mark any where in the arch,

As for EXAMPLE,

At the point C, and then, by a Problem known not only to every Surveyor, but to ordinary Carpenters and Joiners, for finding the center of a circle, whose circumference will pass through three given points that are not in a right line, as ACB, to find the center O. But if you know not how to do it, cross the line AB in the middle as is here done by the perpendicular OC;

OC; so you may by a few trials find both the due extent of your compasses, and the point in your perpendicular, that will fit your purpose near enough; for if a small error be committed in making up the Sector, the most of it goes off again in the subtraction of the triangle.

II. For the latter you may take this ready course; measure the length of both your lines, the chord and the arch, and their distance at the middle of them both; then when you come to protract, first take the length of your right line from the scale, and having laid it down, cross it in the middle at right angles with a dry line, as in the last figure, so shall it intersect the line AB in the point E; then from the same scale, take the measured distance between the 2 lines in the middle, and set it off upon that dry line from the intersection E, to the point C. Then by trials, find a due place in the dry line OEC, and at such a distance with your compasses, that the one foot resting in that line, the other may describe the arch ACB, and the Sector is protracted.

*Concerning customary measure, and how it may be reduced to statute measure,* et e contra, *either by the rule of three, or a more compendious way, by multiplication only.*

Though the statute-perch, or pole, is 16 feet and a half, and no more, yet there are poles of larger measure used in many places, as of 18, 20, 21, 24, and 28 feet, in some 22 and a half. It were therefore very convenient, that our young surveyor were furnished with a chain, fitted to the customary measure of the country where he lives. But because these are too large and cumbersome for small closes, it is very convenient, instead of one chain of 100 links, to make two of 2 poles a piece, each pole divided into 25 links, as that of 100 is; which two half chains may, in the measuring large fields be tied together by the loops with pack-thread, or joined by a buttoning key-ring for more speedy dispatch; but in smaller we may use the half chain of

50 links, only taking care, that we count not half chains for whole ones.

And in these cases where the poles are large, and the closes small, it were still more convenient if you had a chain of two poles only, divided into 100 links. But be sure to remember, when five figures are cut off in the first multiplication, that the content is only roods, and parts of a rood.

We have already observed, that the length of the pole is different in different parts of the kingdom; but there is no occasion for having chains with various sorts of links; since it is very easy to reduce statute measure into customary; and consequently, Gunter's chain will be sufficient in every part of England.

In order to this, it must be remembered, that acres are in proportion to one another, as the square of their poles; and therefore, if you multiply 33, the number of half feet in the statute-pole, by itself, which gives 1089, and also multiply the number of half feet contained in a pole of that measure you would reduce into; in the same manner you may, by the rule of three reverse, obtain your desire, making for that purpose 1089, the first number, the statute measure the second, and the squared half feet of the pole given the third. As for example: Suppose of a close measured by the statute pole, the length and breadth, and their product as are represented in the margin. And it is desired that the content may be cast up according to the large Cheshire measure of 8 yards, or 24 feet to the pole or rood. Then before you cut off any figures, consider that in the statute pole are 33 half feet, and the Cheshire pole 48; multiply therefore 33 by 33, and 48 by 48, and you will have 1089 and 2304, which, together with the said product, may be thus placed; 2304 : 672693 : : 1089, and so multiplying 672693 by 1089, and dividing their product, being 732562677, by 2304, the quotient is 317952; from which, if five figures towards the right hand be cut off, the content, by this customary measure

$$
\begin{array}{rl}
9.33 & \text{L.} \\
7.21 & \text{B.} \\
\hline
933 & \\
1866\phantom{0} & \\
6531\phantom{00} & \\
\hline
672693 & \text{Product.}
\end{array}
$$

ſure of 24 feet to the pole, will be 3 acres, no roods, 28 perches, as appears below:

```
3|17952
      4
-------
 |71808
     40
-------
28|72320
```

But if the lines on the land had been meaſured according to that cuſtom of 24 feet to the pole, and the content muſt have been found according to the ſtatute meaſure, then we muſt have multiplied the product by 2304, and have divided that latter product by 1089. And in the ſame method you may proceed in all, or any of the reſt. But though this way is very exact, plain, and comprehenſive, ſuiting all the cuſtomary meaſures before mentioned, without fractions; it is certainly ſomewhat tedious, except the practitioner knows how to relieve himſelf, by a large table of Logarithms, which we cannot here treat of. Therefore, to contract the work in ſome degree, take notice, that all the cuſtomary poles above-mentioned, except thoſe of 20 and 28 feet, which are rarely uſed, they are each capable of being divided into half yards: And therefore, if inſtead of ſquaring the half feet, you ſquare the half yards of both poles, and work with them, you will attain the ſame end without any material difference, the ſmall diverſity that there is, being generally in the uſeleſs remainders, not in the leaſt affecting the deſired quotient, that gives the anſwer near enough for uſe.

## As for EXAMPLE.

If we had ſquared 11, the number of half yards in the ſtatute pole, which would make 121, and alſo 16, the number of half yards in the Cheſhire pole, which would make 256, and then multiplied the firſt product 672693 by 121, the ſecond product would have been 81819853, which being divided by 256, the quotient would have been, as before, 317952. And this way is, in a manner, co-incident with Mr. Holwell's firſt method.

```
 11
 11
---
 11
 11
---
121

 16
 16
---
 96
16
---
256
```

Take notice alſo, that whether you uſe either of theſe, or the following methods, you need not reduce the particular ſquares, triangles, or trapezia's ſeverally; but ſum up all their products together, and then reduce all at once.

But if you would reduce ſtatute meaſure into cuſtomary, by multiplication, only obſerve the following table:

| The Content by the ſtatute pole, being multiplied by | Gives the content by the pole of | |
|---|---|---|
| .84027 | 18 | Feet. |
| .68062 | 20 | |
| .61734 | 21 | |
| .53777 | $22\frac{1}{2}$ | |
| .47265 | 24 | |
| .34725 | 28 | |

## The Uſe of this TABLE.

When you have multiplied lengths by breadths, or baſes by half perpendiculars, multiply theſe products by the decimal fractions, anſwering to the cuſtomary meaſure, into which you would reduce ſtatute meaſure, and from that latter product, firſt cut off five places towards the right hand, as not to be regarded, being only parts of a ſquare link, then cutting off 5 more, and proceeding to multiply by 4, and then by 40, you will have the content by that cuſtomary meaſure.

EXAMPLE.

### Example.

Suppoſe the length of a cloſe, meaſured by Gunter's chain and multiplied by the breadth, meaſured alſo by the ſame, produced 672693 ſquare links; and it is deſired, that the content may be given in Cheſhire meaſure of 24 feet to the pole; you muſt multiply 672693 by 47265, the decimal fractions anſwering to 24 feet, and from that product, being 31794834645, cut off and caſt away 5 places, and the reſt being 317948, are in the uſual way eaſily reducible into 3 acres, no roods, and 28 perches, which is what it amounted to in the former method exactly.

```
 3|10948
       4
  |71792
      40
28|71680
```

But if you meaſured by a chain of cuſtomary poles, and deſire to know what the content is in ſtatute meaſure, the following table muſt be uſed:

| | | | | |
|---|---|---|---|---|
| The content meaſured by the pole of | 18 | Being multiplied by | 1.19008 | Gives the content by the ſtatute pole. |
| | 20 | | 1.46922 | |
| | 21 | | 1.61983 | |
| | 22½ | | 1.85950 | |
| | 24 | | 2.11570 | |
| | 28 | | 2.87970 | |

To underſtand which, obſerve this Example.

Suppoſe the length and breadth of a long ſquare, being meaſured by a chain of 24 feet to the pole, and multiplied together, make their product 317952, let this be multiplied by 2.11570, which anſwereth to 24 feet, and the latter product will be 67269104640; from which, if you cut off, and caſt away 5 places towards the right hand, the remainder is 672691; which, in the uſual way, is eaſily reduced to 6 acres, 2 roods, and 36 perches.

```
 6|72691
       4
 2|90764
      40
36|30560
```

If the content to be reduced, be caſt up into acres, roods, and perches, reduce all into perches, and then in other reſpects work as before, either by the rule of three, or by this laſt method of multiplication only.

So

So ſhall you have the content in ſquare perches, according to the meaſure deſired, which you may reduce into acres by dividing them by 160; and if any thing remain, that remainder being divided by 40, will give the roods in the quotient, and the latter remainder the number of ſquare perches.

For trial of which rules, obſerve the anſwer of theſe two following queſtions, wrought all three ways.

QUEST. I. How many acres, roods, and perches, according to the pole of 11 feet, are contained in 5 acres, 3 roods, and 11 perches, ſtatute meaſure?

ANSW. Four acres, 3 roods, and 22 perches, as appears by the following work:

FIRST METHOD.

```
                         A. R. P.
   33       36           5  3  11
   33       36           4
  ----     ----         ----
   99      216           23
   99      108           40
  ----     ------       ------
  1089    1296          931
  1089     931          : : 1296
          1089
         ------
          8379
         7448
        9370
        -------
        1013859
       ---------
1296)  1013859  (782
        9072
       ------
        10665                    160)  982  (4
        10368                          640
       -------
         2979                     40)  142  (3
         2592                          120
        ------                        -----
          387                           22
```

## SECOND METHOD.

```
  11     12        144 : 931 : : 121 : 782
  11     12               121
 ----   ----            -------
  11     24               931
  11     12              1862
 ----   ----
 121    144              931
                        -------
                  144) 112651 (782
                        1008
                       ------
                        1185
                        1152
                       ------
                         331
                         288
                        -----
                          43

160) 782 (4  }        { 40) 782 (19
     640     }        {      40
    -----    }   Or   {     -----
 40) 142 (3  }  thus  {     382    4) 19 (4
     120     }        {     360       16
    -----    }        {     -----     ---
     22      }        {      22        3
```

## THIRD METHOD.

```
   84027
     931
  --------
   84027                  160) 782 (4
  252081                       640
 756243                       -----
 ---------                 40) 142 (3
 782.29137                     120
                              -----
                               22
```

QUEST. II. How many acres, roods, and perches of ſtatute meaſure, are contained in 8 acres, 3 roods, or quarters, and 21 perches of 21 feet to the pole?

ANSW. Fourteen acres, 1 rood, and 21 perches, as appears by the three following works in the ſevera methods.

## FIRST METHOD.

```
A. R. P.
 8 : 3 : 21                        42     33
   4                               42     33
 ----                             ----   ----
  35                               84     99
   40                             168     99
 ----                             ----   ----
 1421                             1764   1089

     1089 : 1421 : : 1764 : 2301
              1764
            ------
              5684
             8526
            9947
           1421
           -------
 1089)     2506644   (2301
           2178
           ----
           3286
           3267
           ----
             1944
             1089
           ------
              855
   160)    2301  (14
            160
            ----
            701
            640
          ------
   40)      61  (1
            40
           ----
            21
```

## SECOND METHOD.

```
 14     11      121 : 1421 : : 196 : 2301
 14     11             196
 --     --            ----
 56     11            8526
14     11            12789
---    ---           1421
196    121      ------------
           121)  278516  (2301
                 242
                 ---
                  365
                  363
                  ---
                   216
                   121
                ------
           160)  2301  (14
                 160
                 ----
                  701
                  640
                  ---
                   61
             40)   61  (1
                   40
                   --
                   21
```

## THIRD METHOD.

```
    1. 61983          160)  2301  (14
        1421                160
  ------------              ----
      161983                 701
     323966                  640
    647932                  -----
   161983              40)   61  (1
  ------------                40
  2301.77843                  --
                              21
```

*How to measure a piece of land, with any chain, however divided; or even with a cord, or cart-rope.*

If you can procure a chain, and find it is not divided in the manner we have already explained, but into feet, or quarters of yards, or any such vulgar divisions, make no reckoning of the divisions, but measure it as exactly

as you can, to find out the true length of the whole chain; and if it fit none of thoſe lengths before-mentioned, nor any of their halfs, make it fit by taking off a link or two, or piecing it out with a ſtring; then dividing the length of that chain by 100, or the half of it by 50, find the true length of a link, according to our artificial diviſions, and having got a long ſtick, or rod, ſet as many of thoſe link-lengths upon it as it will hold; then may you meaſure all the whole chains by your regulated chain, and the odd links of every line, by your divided ſtick, or rod, as will be proved by the following example.

Being far from any inſtrument, and requeſted by a friend to meaſure him a cloſe, and you can procure a pair of compaſſes, an ordinary carpenter's rule of two foot, divided into inches and quarters, and alſo a piece of an old chain, ſeemingly divided into feet, you muſt meaſure it by the rule, and finding it to be 45 feet long, and ſome odd meaſure, piece it out with a ſtrong cord that will not ſtretch, to 48 feet exactly; then it will ſerve for half a chain of 24 feet to the pole: This 48 multiply by 12, the number of inches in a foot, and that product being 576, divide it by 50, the number of links in half a decimal chain, and the quotient is $11\frac{66}{100}$ inches, or 11 inches and a half, and ſomething over. Dividing therefore a long ſtick throughout into ſuch parts, each containing 11 inches and a half, beſides the breadth of the nicks, you are provided with tools to meaſure lines to a link with ſufficient exactneſs.

In like manner you may proceed with a cord or rope, having fitted them to ſome known length; and then for protracting, it is eaſy with the compaſſes to make a plain ſcale of a large ſort, either upon paper, or an even piece of wood.

Beſides, there is a way of meaſuring the perpendiculars of triangles and trapezias upon the ground itſelf, ſo as to prevent the neceſſity of a ſcale; for if you have a ſmall ſquare with a hole in it, to turn upon the head of a ſtick, which you may fix where you pleaſe, as you are meaſuring the baſe of a triangle, or the diagonal of a trapezium; you may by a very few trials find

the place where the one leg will be juſt in the line you are meaſuring, and the other point at the angle from which the perpendicular falls on it, and then the ſpace between your ſtick and that angle truly meaſured, is the perpendicular.

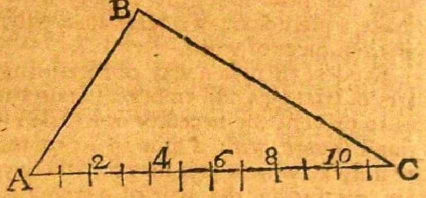
If you have not ſuch a ſquare, a ſquare trencher, or any end of a board that hath one right angle, and two true ſides, will ſupply the want of it. This is undoubtedly an excellent way of meaſuring a trapezium, though it be protracted afterwards; for by meaſuring the perpendiculars as aforeſaid, and obſerving at how many chains and links end, the ſaid perpendiculars meet the common baſe, the whole trapezium may be truly protracted without going about it; this little ſquare ſufficiently ſupplying the place of an inſtrument, which is uſually called a croſs or ſquare, made up, as it were, of two ſmall indices, like thoſe for a plain table, but much leſs, with fore-ſights and back-ſights, and cutting one another at right angles put together, and having an hole at the center.

*Concerning dividing of land artificially and mechanically.*

I. To divide a triangle into any parts required; divide the baſe as the demand imports, then ſhall lines drawn from the points of diviſion to the oppoſite angle, finiſh the diviſion of the triangle.

EXAMPLE.

AC, the baſe of the triangle ABC, being divided into 12 equal parts, a line drawn from the angular point B to the point 6, divides the triangle into two equal parts; two lines drawn to 4 and 8, divide it into three equal parts; 2 lines drawn




 to

to the points marked with 3, 6, and 9, divide it into four equal parts; and so lines drawn to 2, 4, 6, 8, 10, divide it into six equal parts.

It is likewise very obvious, that if the same triangle were to be so divided, that the one part should be double to the other, a line drawn from B to 4 or 8, doth the work, or if it be required to divide it into two parts, so as the one shall be triple to the other, a line drawn from B to 3 or 9, compleats the work. In like manner a line from B to 2 or 10 divides it into two parts, whereof the one is quintuple, or five-fold, to the other, and a line from B to 1 or 11, divides it into two parts, whereof the one is 11 times as large as the other.

Farther yet, if it were required that this triangle should be so divided, that the two parts should in quantity bear proportion, as 5 and 7, a line from B to 5 or 7, will answer the purpose.

The divisions will, indeed, be sometimes a little more intricate than this, yet not such, but that the seeming difficulty may be easily overcome, by observing the following method.

Suppose, a large triangle of common land to be divided amongst three tenants, A, B, and C, according to the quantity of their tenements: A, having 19 acres of land to his tenement, B 13, and C 7, the base of the triangle being found by measure to be 17 chains 27 links: and the demand is, Where the point of the division must be placed in the base, so as lines drawn from thence to the opposite angle, shall truly limit each man's part?

To answer this, let us add 13 and 07 to 19, and they give 39: so is the work plainly reduced to the rule of fellowship; and therefore, to find every man's distinct portion, we need only to multiply the base by his particular number, and divide that product by 39, the sum of all their numbers.

| |
|---|
| 19 |
| 13 |
| 07 |
| 39 |

A.

```
39 : 17. 27 : : 19
              19
        ---------
         15543
         1727
       ---------
39) 328.13 (8.41 14/39
     312
    -----
      161
      156
    -----
       53
       39
     ----
       14
```

B.

```
39 : 17. 27 : : 13
              13
        ---------
          5181
          1727
       ---------
39)  224.51 (5.75 26/39
      195
     -----
      295
      273
     -----
      221
      195
     -----
       26
```

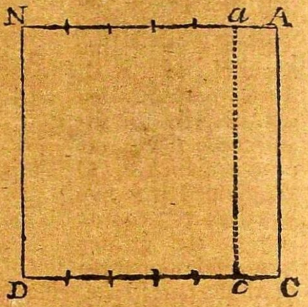
C.

```
39 : 17. 27 : : 7
              7
         --------
39)      120.89 (3.09 18/39
          117
         -----
          389
          351
         -----
           38
```

From these operations it is plain, that if we ſet off from the angular point where the baſe begins, 8 chains 41 links, and a little above the third part of a link upon the baſe for A, and where that ends, 5 chains and 75 links and ½ of a link for B, and conſequently, leave between this ſecond diviſion, and the other end of the baſe, 3 chains and almoſt 10 links, for C; lines drawn from thoſe points of diviſion to the oppoſite angle, will give each man his due.

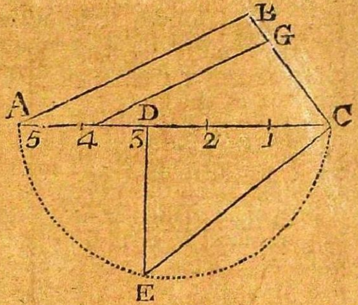
What has been ſaid, reſpecting the diviſion of triangles upon their baſes, will, with a little variation, ſerve for the dividing of all ſorts of Parallelograms, either ſquares, long ſquares, Rhombus's, or Rhomboides; all the difference is, that inſtead of drawing lines from points in the baſe to the oppoſite angle, you muſt draw parallel lines from the points in one oppoſite ſide to another, as will be ſufficiently plain by the following inſtance.

Suppose the square figure to represent a close of six acres, and you are to cut off an acre at the side AC; having set off the 6th part of the line CD, from C towards D, as at *c*; also the sixth part of the line AN from A towards N as at *a*: draw the line *ca* parallel to AC, and this takes off exactly a 6th part of an acre; that is, the small Parallelogram A C *c a*, is one acre.



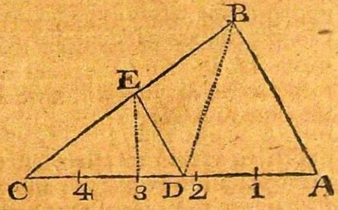

If it be not thought convenient, as in some cases it is not, to cut off a piece so long and narrow, you may, by the rule of three, find what other length of any greater breadth, will contain an equal quantity. Or you may multiply the breadth by 2, 3, or any other number, and divide the length by the same number that you multiplied the breadth by. Or lastly, if you set off a double proportion, that is $\frac{2}{6}$ or $\frac{1}{3}$ from C towards D, and from the point where it falls, draw a line to the angle A, you will have a triangle equal to $\frac{1}{6}$ of the square ACDN.

But to return to triangles, the most simple and primitive of all Rectilinears, and therefore, the most considerable in this case of partition, as giving laws often to the rest; it may fall out, that a triangle must be divided, by a line from some point in a side, so as that line may either be parallel to some other side, or not parallel to any.




Let ABC be a triangle given, and let it be required to cut off $\frac{3}{5}$ by a line parallel to A B. First, on the line AC describe the semi-circle AEC, on whose diameter, C A, divided into five equal parts according to the greater term, and upon three of those parts, the lesser term, erect the perpendicular DE, which cuts the arch line in E, then set the length of the line CE from C to 4, and from thence draw the line 4 G parallel to AB; so will the triangle CG 4 contain $\frac{3}{5}$ of the triangle ABC, as was required.

Now, for the latter case, when the line of partition goes not parallel with any side, take this example.




Let ABC be a triangle given to be divided into two parts which shall be in proportion to one another, as 3 is to 2, by a line drawn from the point D in the base, or line AC.

From the limited point D, draw a line to the angle B; then divide the base AC into five equal parts, and from the third point of the division draw a line to E parallel to BD. Lastly, from E draw the line ED, so shall the trapezium ABED be in content as 3 to 2, to the new triangle DEC.

Before we have done with the division of triangles, it will be necessary to add the three following cautions.

1. You must be sure to take very exactly the distance of every point, where a division line cutteth any side; to one of the ends of the same side, as in this last figure, the distances BE and AD; which distances be-

ing applied to the ſcale by which the triangle was protracted, will ſhew at how many chains and links end you are to make your diviſion or line on the field itſelf.

2. The proportions by which you are to divide, are not always ſo formally given as in the former examples, but are ſometimes to be found out by arithmetical working; as in this caſe.

Suppoſe a triangular field of 6 acres, 2 roods, and 31 perches, muſt be divided ſo that one of the two parts ſhall be 4 acres, 3 roods, and 5 perches, and the other conſequently 1 acre, 3 roods, and 26 perches; reduce both meaſures into perches, and the one will be 705, and the other 306. Their ſum is 1701, which by their common meaſure, being reduced into their loweſt terms of proportion in whole numbers, will be 5, 2, and 7, which ſhews that the triangle being divided into 7 equal parts, the one muſt have 5 of thoſe 7 parts, and the other 2. And obſerve, that it will be ſufficient to find the common meaſure between the ſum of your terms, and either of the terms; the method whereof is ſhewn in every arithmetical book for reducing fractions to their loweſt terms.

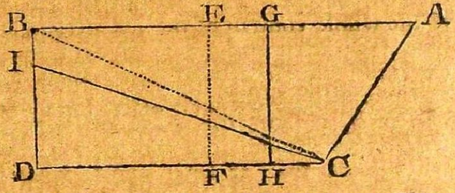
But you may multiply either of the parts; as ſuppoſe 765, by the length of the baſe, which we will ſuppoſe to be 8 chains and 75 links, or 87 links; and that product divided by 1071, the content of the whole cloſe in perches, gives by the rule of three direct, 625 links, or 6 chains and 1 pole, the true diſtance from either end of the baſe, that your mind or occaſions may direct you to the point of diviſion; for the diviſion muſt not only be for proportion or quantity, but alſo as to poſition or ſituation of parts upon the paper, as it is required to be on the ground.

3. In theſe and all other diviſions of land, where a ſtrict proportion in quantity is to be obſerved, you muſt have a reſpect to the rules given hereafter concerning the meaſuring of uneven ground, eſpecially if one part prove much more uneven than another; and if there be any uſeful pond or well, you muſt draw your line of diviſions through it; but if it be an uſeleſs pond, lake, or puddle, or if there be any boggy or barren ground, that

that muſt be caſt out in the diviſions; meaſure that firſt, and ſubtract it from the content of the whole cloſe, and then lay the juſt proportion of the remainder on that ſide that is free from it, that the other may have its juſt part alſo, beſides that which is uſeleſs.

What hath been ſaid, with an ordinary meaſure of diſcretion, may ſufficiently inſtruct a young artiſt to divide triangles, Parallelograms, and regular Polygons, in an artificial way; but becauſe many cloſes and open grounds are Trapezias, and many irregular Polygons, and even thoſe that are regular enough, may fall under an irregular diviſion, in regard of the quality of the land, woods upon it, or quarries in it, or the conveniencies of ways, currents of water, ſituation in reſpect to adjacent lands, &c. we ſhall propoſe a method, which though it has ſomewhat of the mechanic in it, will be ſingularly uſeful in ſuch caſes.

Let A B DC be a Trapezium to be divided betwixt a young heir and his mother, ſo that




his part may be double to her's. Having by a Diagonal BC divided it into two triangles, you will find the content of the triangle ABC to be 138562 ſquare links, and the triangle BCD to contain 103468, in all 242020 ſquare links, which, if reduced, as hath been formerly taught, would amount to 2 acres, 1 rood, and 27 perches.

But for the preſent work, they are in a better order already. Dividing then 242020 into three parts, each of them is 80676 and $\frac{2}{3}$, two thirds of theſe third parts muſt contain 161353$\frac{1}{3}$; but the fraction $\frac{1}{3}$ being inconſiderable, regard it not.

Then if you reſolve to lay out the double part towards the line BD, ſtrike at a venture the line EF, and meaſure the trapezium bounded by that line, and the oppoſite ſide BD, together with the interjacent parts of the lines AB and CD, which you will find to contain

tain 119140 ſquare links; but becauſe it ſhould have been 161353, ſubtract 11940 out of 161353, and their difference is 42213, and perceiving that the lines AB and CD are very nearly parallel, and finding their diſtances where they are cut by the line EF, to be 326 links, or 3 chains and 26 links, divide 42213 by 326, and the quotient is 129 links and almoſt a half, at which diſtance draw the line GH parallel to EF, ſo ſhall the trapezium GBDH be the heir's part.

Another way whereby the ſame thing may be performed is this. Finding the triangle ABC to contain 138650 ſquare links, ſubtract it out of the heir's part, viz. 161353, the difference 22803, ſhews how many ſquare links muſt be taken out of the triangle BCD, and added to the ABC; which to perform with all neceſſary exactneſs, ſuppoſe the ſide or line BD to be the baſe, which by meaſure proves to be 344 links, or 3 chains, and 44 links. Say by the rule of three direct, if the whole content of the leſſer triangle, viz. 103468, give 344; what will 22803 give? So will the reſult be 75 links, and ſomewhat more than $\frac{4}{5}$ of a link, for 22803 multiplied by 344, gives 7844232, which being divided by 103468, the quotient is $75\frac{182132}{003468}$, or according to decimal diviſion, 75.8131, which is ſomewhat more than 75 links and $\frac{4}{5}$, wherefore extending your compaſſes upon the ſcale to almoſt 76 links, ſet that diſtance upon the line BD, from B to I, and draw the line CI; ſo ſhall the trapezium ABIC be double to the triangle ICD, within ſo ſmall a matter, as is not worth regarding, though the land were a rich meadow.

*Concerning the boundaries of land, or where the lines to be meaſured, muſt begin and end.*

If there be no agreement between the parties concerned, (for if there be, that muſt be obſerved) reaſon and cuſtom are the Surveyor's guide.

The farmer ſpeaks loudly, that when a piece of arable or meadow land is to be ſown or mown, no more ſhould be meaſured, nor expected to be paid for, either to the letter or workmen, than the plough or ſcythe can go

go over. So alſo when a parcel of land is let for paſture to a farmer, it ſeems very reaſonable that all, and only ſo much ſhould be meaſured, as is uſeful for that purpoſe.

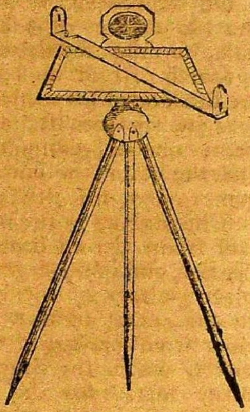
But commons to be incloſed, are uſually meaſured, except it be otherwiſe agreed, to the uttermoſt bounds of every man's particular proportion, without any allowance for ditch or fence, every man being to make them upon his own of what breadth he pleaſes. Nor is this unreaſonable, for it is as good for one as another, and the rate paid to the Lord is uſually very ſmall, and ſometimes nothing.

It is alſo very uſual in meaſuring betwixt lord and tenant, in caſe of leaſes for lives or long terms of years, to extend the lines to the utmoſt bounds of the tenant's claim, taking in the walls, hedges, and ditches; but this is accounted very hard, and frequently proves very unequal among the tenants of a lordſhip; ſome being forced to make much more waſte of their ground this way, than others that hold as much or more. But where the cuſtom obtains, the ſurveyor muſt obſerve it; for it is the buſineſs of others to appoint what muſt be meaſured, and he is only to meaſure truly what is ſo appointed.

Laſtly, in caſe of a ſale by meaſure, at a rate agreed upon by each, no boundaries being ſpecified in the bargain, the rule is, to extend the lines to the quick-wood-row, that is, to the place where the quick-wood actually grows, or where, according to the general rule, it ought to be ſet.

*A deſcription of the plain table, the protractor, and lines of chords.*

Though what hath been already ſaid, may competently ſuffice to inſtruct any young artiſt in meaſuring a cloſe of land, yet to advance him a degree higher in uſeful knowledge, we ſhall take occaſion to deſcribe to him the plain table, which conſiſts of ſeveral parts.



The table itself is a Parallelogram of oak or any other wood, about fifteen inches long, and twelve broad, consisting of two several boards, round which are ledges of the same wood; the two opposite of which being taken off, and the spangle unscrewed from the bottom, the aforesaid two boards may be taken asunder for ease and convenience of carriage. For the binding of the two boards and ledges fast, when the table is set together, there is a box-jointed frame, about three quarters of an inch broad, and of the same thickness as the boards, which may be folded together in six pieces. This frame is so contrived, that it may be taken off and put on the table at pleasure, and may go easily on the table either side being upwards. This frame also is to fasten a sheet of paper upon the table, by forcing down the frame, and squeezing all the edges of the paper, so that it lies firm and even upon the table, that thereby the plot of a field or other inclosure, may conveniently be drawn upon it.

On both sides this frame, near the inward edge, are scales of inches subdivided into ten equal parts, having their proper figures set to them. The uses of these scales of inches are for the ready drawing of parallel lines upon the paper; and also for shifting your paper, where one sheet will not hold the whole work.

Upon one side of the said box-frame, are projected the 360 degrees of a circle, from a brass center hole in the middle of the table. Each of these degrees are subdivided into thirty minutes; to every tenth degree are set two numbers, one expressing the proper numbers of degrees, and the other the complement of that number of degrees to 360. This is done to avoid the trouble of subtraction in taking of angles.

On the other ſide of this frame, are projected the 180 degrees of a ſemi-circle from a braſs center hole, in the middle of the table's length, and about a fourth part of its breadth. Each of theſe degrees are ſubdivided into thirty minutes; to every tenth degree are ſet likewiſe, as on the other ſide, two numbers; one expreſſing the proper number of degrees, and the other the complement of that number of degrees to 180, for the ſame reaſon as before.

The manner of projecting the degrees on the aforeſaid frame is, by having a large circle divided into degrees, and every degree into thirty minutes; for then placing either of the braſs center holes on the table, in the center of that circle ſo divided, and laying a ruler from that center to the degrees on the limb of the circle, where the edge of the ruler cuts the frame, make marks for the correſpondent degrees on the frame.

The degrees thus inſerted on the frame, are of excellent uſe in wet or ſtormy weather, when you cannot keep a ſheet of paper on the table. Theſe degrees will alſo make the plain-table a Theodolite, or ſemi-circle, according to which ſide of the frame is uppermoſt.

There is a box, with a needle and card, covered with a glaſs, fixed to one of the long ſides of the table, by means of a ſcrew, that it may be eaſily taken off. This box and needle is very uſeful for placing the inſtrument in the ſame poſition upon every remove.

There belongs to this inſtrument a braſs ſocket and ſpangle, ſcrewed with three ſcrews to the bottom of the table, into which muſt be put the head of the three-legged ſtaff, which may be ſcrewed faſt by means of a ſcrew in the ſide of the ſocket.

There is alſo an index belonging to the table, which is a large braſs-ruler, at leaſt ſixteen inches long, and two inches broad, and ſo thick as to make it ſtrong and firm having a ſloped edge called the fiducial edge, and two ſights ſcrewed perpendicularly on it of the ſame height. They muſt be ſet on the ruler exactly at the ſame diſtance from the fiducial edge. Upon this

this index it is uſual to have many ſcales of equal parts, as alſo Diagonals and lines of chords.

When you would make your table fit for uſe, lay the two boards together, and alſo the ledges at the ends in their due places, according as they are marked. Then lay a ſheet of white paper all over the table, which muſt be ſtretched over the boards, by putting on the box-frame which binds both the paper to the boards, and the boards to one another: then ſcrew the ſocket on the back-ſide of the table, and alſo the box and needle in its due place, the meridian line of the card lying parallel to the meridian of the diameter of the table; which diameter is a right line drawn upon the table from the beginning of the degrees through the center, and ſo to the end of the degrees. Then put the ſocket upon the head of the ſtaff, and there ſcrew it. Put alſo the ſights upon the index, and lay the index upon the table. So is your inſtrument prepared for uſe, as a plain table, Theodolite, or ſemi-circle.

The Protractor is an inſtrument ſo well known, and ſo eaſy to be made and procured, that we ſhall be very brief in the deſcription of it. As it is uſually made, it conſiſts of two parts, a ſcale and a ſemi-circle; the ſcale is however no neceſſary part of it, but ſerves if you are not otherwiſe provided, for uſes before-mentioned in the caſe of the plain ſcales.

But the ſemi-circle is more eſſential. This may be made of braſs, or other metal; and of any convenient ſize, as four inches, more or leſs, for the ſtreight ſide; this ſemi-circle being bounded as all others are by two lines; the one right or ſtreight, the other circular.

The right line is divided preciſely in the middle, by a point which is in the center, upon which the circular boundary is drawn, and two other arches concentrical with it.

The center, when the ſemi-circle goes alone without the ſcale, ſhould be guarded with two lips, one on each ſide, or a little loup, for the conveniency of turning the inſtrument about a pin fixed in a paper.

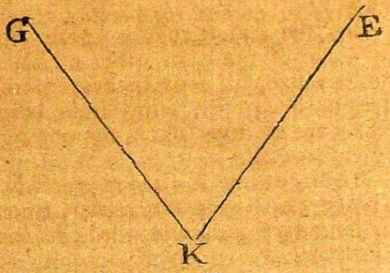
The

The arched or circular edge is divided into 180 degrees, or equal parts, numbered by tens, upon the upper concentric arch from 0 to 180, and in the lower from 180 to 860. So that by applying the ſtreight edge of the Protractor twice to any line, keeping the center right upon a pin fixed in the line, that is, with the ſemicircle firſt above it, and then below it, or contrarily, you may draw a whole circle by the guidance of the arch, or ſet out any number of degrees, as will appear more plainly hereafter.

A line of chords is a line divided into 90 unequal parts, whereof 60, and the Radius upon which the circle was drawn, are equal, and the diviſions upon that line are equal to the next extent in a right line of ſo many degrees from the beginning of the quadrant, as anſwer thereto.

When lines of chords are cut upon wood, it is both uſual and neceſſary that there be two ſtuds of braſs, the one at the beginning, and the other at 60 degrees, with little Holes for the feet of the compaſſes, when you take the extent of the Radius, to preſerve the line from being wounded by the compaſſes, and thus fenced, it will, in caſe of neceſſity, do the work of a Protractor, but not altogether ſo commodiouſly.

1. *How to obſerve an angle in the field by the plain table.*




Suppoſe E, K, G, to be two hedges, or two ſides of a field, including the angle EKG, and it is required to draw upon the table an angle equal thereto: Firſt place your inſtrument as near the angular point K as conveniency will permit, turning it about till the north end of the needle hang directly over the meridian line in the card, and then ſcrew the table faſt. Then upon your paper, will

with your protracting pin, which is a fine needle put into a piece of box, or ivory, neatly turned, or compass-point, assign any point at pleasure upon the table, and to that point apply the edge of the index, turning the index about upon that point, 'till through the sights you see a mark set up at E, or parallel to the line E K, and then with your protracting-pin, compass-point, or pencil, draw a line by the side of the index to the assigned point upon the table: then the table remaining immoveable, turn the index about upon the fore-mentioned point, and direct the sights to the mark set up at G, or parallel thereto, that is, so far distant from G, as your instrument is placed from K: and then by the side of the index, draw another line to the assigned point. Thus will there be drawn upon the table two lines representing the hedges EK, and KG, and which include an angle equal to the angle EKG. And though you know not the quantity of this angle, yet you may find it, if required: for in working by this instrument, it is sufficient only to give the proportions of angles, and not their quantities in degrees, as in working by the Theodolite, semi-circle, or circumferentor. Also in working by the plain table, there needs no protraction at all, for you will have upon your table the true figure of any angle or angles that you observe in the field in their true positions, without any further trouble.

2. *How to find the quantity of an angle in a field, by the Plain Table, considered as a Theodolite or semicircle.*

Let it first be required to find the quantity of the angle EKG (see the above figure) by the plain table, as a Theodolite. Place your instrument at K, with the Theodolite side of the frame upwards, laying the index upon the diameter thereof; then turn the whole instrument about, the index still resting upon the diameter, till through the sight you espy the mark at E: then screwing the instrument fast there, turn the index about upon the Theodolite center hole, in the middle of the table, till thro' the sight you espy the mark at G. Then note what degrees on the frame of the table are cut by the

the index, and thoſe will be the quantity of the angle EKG ſought.

You muſt proceed in the ſame manner for finding the quantity of an angle by the plain table as a ſemi-circle; only put the ſemi-circle ſide of the frame upwards, and move the index upon the other center hole.

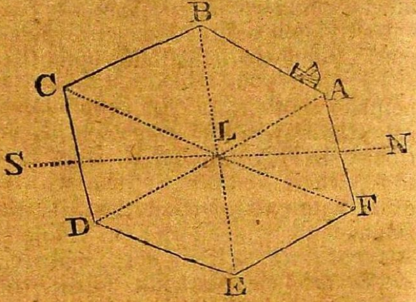
3. *How by the plain table, to take the plot of a field at one ſtation within the ſame; from whence all the angles of the ſame field may be ſeen.*

Having entered upon the field to ſurvey, your firſt work muſt be to ſet up ſome viſible mark at each angle thereof; which being done, make choice of ſome convenient place about the middle of the field, from whence all the marks may be ſeen, and there place your table, covered with a ſheet of paper, with the needle hanging directly over the meridian line of the card, which you muſt always have regard to, eſpecially when you are to ſurvey many fields together. Then make a mark about the middle of the paper, to repreſent that part of the field where the table ſtands; and laying the index upon this point, direct your ſights to the ſeveral angles where you before placed marks, and draw lines by the ſide of the index upon the paper. Then meaſure the diſtance of each of theſe marks from your table, and by your ſcale ſet the ſame diſtances upon the lines drawn upon the table, making ſmall marks with your protracting-pin, or compaſs-point, at the end of each of them. Then lines being drawn from one to the other of theſe points, will give you the exact plot of the field; all the lines and angles upon the table being proportional to thoſe of the field.

EXAMPLE.

## EXAMPLE.

Suppoſe the plot of the field ABCDEF was to be taken. Having placed marks in the ſeveral angles thereof, make choice of ſome proper place about the middle of the field, as at L, from whence you can ſee all the marks before placed in the ſeveral angles; and there place your table. Then turn your inſtrument about, till the needle hang over the meridian line of the card, denoted by the line NS.




Your table being thus placed with a ſheet of paper thereon, make a mark about the middle of your table, which ſhall repreſent the place where your table ſtands. Then applying your index to this point, direct the ſights to the firſt mark at A; and the index reſting there, draw a line by the ſide of your index to the point L. Then with your chain meaſure the diſtance from L, the place where your table ſtands to A, the firſt mark, which ſuppoſe eight chains ten links. Then take eight chains ten links from any ſcale, and ſet that diſtance upon the line from L to A.

Then directing the ſights to B, draw a line by the ſide of the index as before, and meaſure the diſtance from your table at L, to the mark at B, which ſuppoſe eight chains, ſeventy-five links. This diſtance taken from your ſcale, and being applied to your table from L to B, will give the point B, repreſenting the ſecond mark. Then direct the ſights to the third mark C, and draw a line by the ſide of the index, meaſuring the diſtance from L to C, which ſuppoſe ten chains ſixty-five links. This diſtance being taken from your ſcale,

and applied to your table from L to C, will give the point C, representing the third mark.

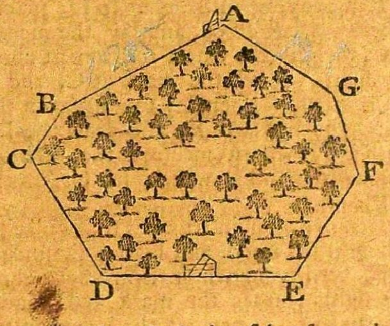
In this manner you must deal with the rest of the marks at D, E, and F, and more if the field had consisted of more sides and angles.

Lastly, when you have made observations of all the marks round the field, and found the points ABCDE and F upon your table, you must draw lines from one point to another, till you conclude where you first began; as draw a line from A to B, and from B to C, from C to D, from D to E, from E to F, and from F to A, where you began; then will ABCDEF be the exact figure of your field, and the line NS, the meridian.

4. *To take the plot of a wood, park, or other large level plain by the plain table, in measuring round the same.*

Suppose ABCDEFG to be a large wood, whose plot you desire to take upon the plain table.

Having put a sheet of paper on the table, place your instrument at the angle A, and direct your sights to the next angle at B, and by the side of the index draw a line upon your table, as the




line AB. Then measure by the hedge side, from the angle A to the angle B, which suppose twelve chains, five links. Then from your scale take twelve chains, five links, and lay off upon your table from A to B. Then turn the index about, and direct the sights to G, and

and draw the line AG upon the table. But at preſent you need not meaſure the diſtance.

Remove your inſtrument from A, and ſet up a mark where it laſt ſtood, and place your inſtrument at the ſecond angle B. Then laying the index upon the line AB, turn the whole inſtrument about, till through the ſights you ſee the mark ſet up at A, and there ſcrew the inſtrument. Then laying the index upon the point B, direct your ſights to the angle C, and draw the line BC upon your table. Then meaſuring the diſtance BC four chains, forty-five links, take that diſtance from your ſcale, and ſet it upon your table from B to C.

Remove your inſtrument from B, and ſet up a mark in the room of it, and place your inſtrument at C, laying the index upon the line CB; and turn the whole inſtrument about, till through the ſights you eſpy the mark ſet up at B, and there faſten your inſtrument. Then laying the index on the point C, direct the ſights to D, and draw upon the table the line CD. Then meaſure from C to D eight chains, eighty-five links, and ſet that diſtance upon your table from C to D.

Remove your inſtrument to D, placing a mark at C, where it laſt ſtood, and lay the index upon the line DC, turning the whole inſtrument about, till through the ſights you ſee the mark at C, and there faſten the inſtrument. Then lay the index on the point D, and direct the ſights to E, and draw the line DE. Then with your chain meaſure the diſtance DE thirteen chains four links, which lay off on the table from D to E.

Remove your inſtrument to E, placing a mark at D, where it laſt ſtood, and laying the index upon the line DE, turn your whole inſtrument about, till through the ſights you ſee the mark at D, and there faſten the inſtrument. Then lay the index on the point E, and direct the ſights to F, and draw the line EF. Then meaſure the diſtance EF, ſeven chains ſeventy links, which take from your ſcale, and lay off from E to F.

Remove your inſtrument to F, placing a mark at E, where it laſt ſtood, and lay the index upon the line EF, turning the inſtrument about, till you ſee the mark ſet up at

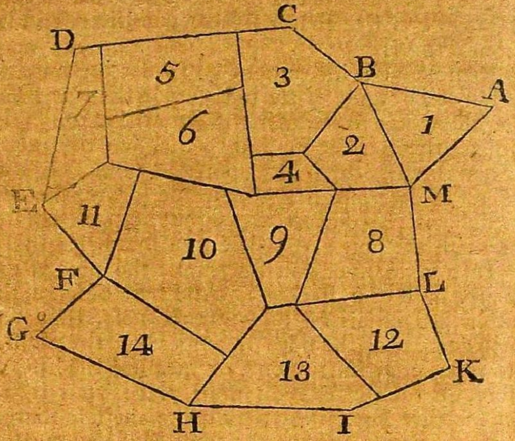
at E, and there faſten the inſtrument. Then laying the index on the point F, direct the ſights to G, and draw the line FG upon the table, which line FG will cut the line AG in the point G. Then meaſure the diſtance FG five chains ſixty-ſeven links, and lay it off from F to G.

Remove your inſtrument to G, ſetting a mark where it laſt ſtood, and lay the index upon the line FG, turning the whole inſtrument about, till through the ſights you ſee the mark at F, and there faſten the inſtrument. Then laying the index upon the point G, direct the ſights to A, your firſt mark, and draw the line GA, which, if you have truly wrought, will paſs directly through the point A, where you firſt began.

In this manner you may take the plot of any level plain, be it ever ſo large. And here obſerve, that very often hedges are of ſuch a thickneſs, that you cannot come near the ſides or angles of the field, either to place your inſtrument, or meaſure the lines: therefore, in ſuch caſes, you muſt place your inſtrument, or meaſure your lines parallel to the ſide thereof; and then your work will be the ſame as if you meaſured the hedge itſelf. Take notice alſo, that in thus going about the field you may aſſiſt yourſelf greatly by the needle. For looking what degree of the card the needle cuts at one ſtation, if you remove your inſtrument to the next ſtation, and with your ſights look to the mark where your inſtrument laſt ſtood, you will find your needle to cut the ſame degree again, which will give you no ſmall ſatisfaction in the proſecution of your work. And though there be a hundred or more ſides, the needle will ſtill cut the ſame degrees at all of them, except where you have committed ſome former error; therefore, at every ſtation have an eye to the needle.

*Concern-*

*Concerning the method of plotting many closes together, whether the ground be even or uneven.*



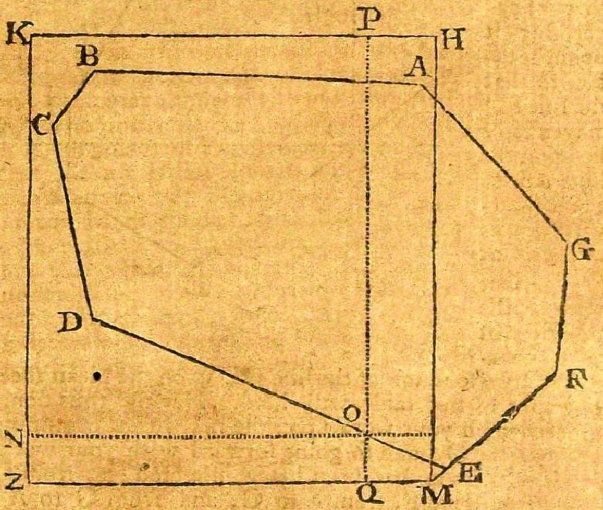

Suppose A B C D E F G H I K L M to be a tenementor smalldemesne, divided into 14 closes, to be measured and protracted according to their several shapes and situation: First, draw the plot of the whole by the method of circulation, planting your instrument either at every angle, or only at the most considerable, either within or without, as you find most convenient. This being done, a line from B to M, gives the triangle ABM for the first close. In the next place go round the second close, beginning at M, then to B, and so about to M again; then for the third close, plant your table at C, and go round to B, the line CB being already protracted, and so of all the rest, still observing which are common lines belonging to several closes, representing the fences, that you may avoid the trouble of measuring those lines oftner than once, and lay every part of every close in its due place; and be sure to keep the instrument throughout the whole work in its true position by needle, fore-sight, and back-sight.

There are divers other ways of performing this work, but none more sure or plain, especially if the ground be uneven; for in that case, if you protract accord-

according to the length of lines meaſured from your ſtation to the angles, you will put your cloſes into unproportionable ſhapes, except you reduce hypothenuſal lines to horizontal, by inſtruments, or otherwiſe, which is ſomewhat troubleſome; and the like may be ſaid when you plot with the chain only.

*Concerning ſhifting the Paper.*

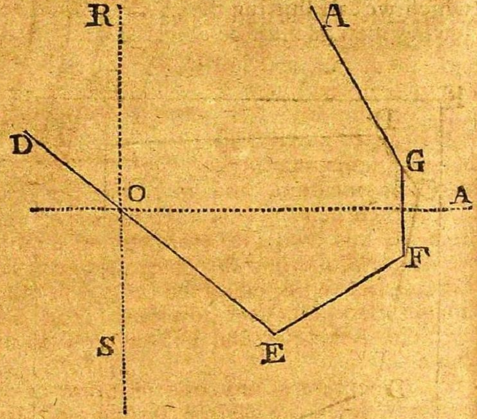
In taking the plot of a field by the plain table, and going about the ſame, as before directed, it may ſo happen when the field is remarkably large, or when you are to take many incloſures together, that the ſheet of paper on the table will not hold all the work. In this caſe you muſt be obliged to take off that ſheet, and put another clean one in the room of it; and in plotting a manor or lordſhip, many ſheets may be thus changed which we call ſhifting the paper. To do this, the following method muſt be obſerved.



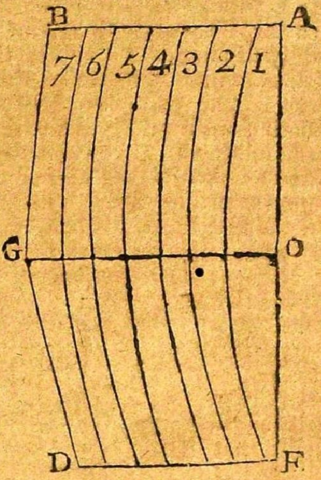

Suppoſe in going about to take the plot ABCDEFG as before directed, that you make choice of the angle

at A for the place of beginning, and proceed from thence to B, from B to C, and from C to D; when you come to the angle at D, and are to draw DE, you want room to draw the ſame upon the table; do thus:

Firſt, Through the point D draw the line DO, which is almoſt as much of the line DE as the table will contain. Then near the edge of the table HM, draw a line parallel to HM, by means of inches and ſubdiviſions on the oppoſite ſides of the frame, as PQ, and another line at right angles to that through the point O, as ON. This being done, mark this ſheet of paper with the figure (1) about the middle thereof, for the firſt ſheet. Then, taking this ſheet off your table, put another clean ſheet thereon, and draw upon it a line parallel to the contrary edge of the table, as the line RS. Then taking your firſt ſheet of paper, lay it upon the table, ſo that the line PQ may exactly lie upon the line RS, to the beſt advantage, as at the point O. Then with the point of your compaſſes draw ſo much of the line OD upon the clean ſheet of paper as the table will hold. Having thus done, proceed with your work upon the new ſheet, beginning at the point O; and ſo going forward with your work, in all reſpects as has been before directed; as from O to E, from E to F, from F to G, and from G to A; ſhifting your paper as you have occaſion.



*Concerning the plotting of a town-field, where the ſeveral lands, butts, or doles, are very crooked; with an obſervation concerning hypothenuſal, or ſloping boundaries.*



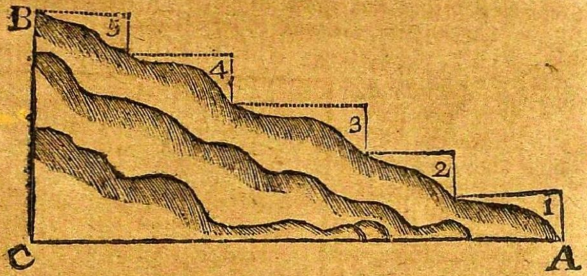

Suppoſe ABGDFO divided, in the manner of a common field, into parts or doles, belonging to ſeven ſeveral men: firſt plot the whole, as hath been before directed; then meaſuring from A to B upon the land, note down, as you go along, at how many chains or links, or both, the diviſion is between dole and dole, and accordingly mark them out by the help of ſcale and compaſſes on the line AB on the paper-plot. In the very ſame manner you muſt meaſure, and mark out the lines OG and FD; which being done, take the paper from the inſtrument, and laying it before you on a table, with the ſide AF towards you, the compaſſes muſt be ſo placed, that one foot reſting upon the table, the other may paſs through the points of diviſion upon all the three lines, viz. AB, OG, and FD, as they do in the above figure.

If the content of any one or more of theſe parts, butts, or doles, be deſired without plotting, it may eaſily be found, by your plain table, in the manner following.

Take the breadth by your chain at the head and lower end, and adding thoſe numbers together, the third part of the ſum is the equated breadth; by which multiplying the length meaſured down the edge, or middle, the product gives the content.

But both in this caſe and that mentioned before, the figure of a plot may be ſomewhat diſordered, not only

the unevenneſs of the ground within, but alſo by the greater declivity of the ground where the boundary lines paſs, either of the whole plot, or particular parcels. For whereas in plotting, every line is preſumed to be horizontal, or level, that it may paſs from angle to angle the ſhorteſt way, and that every part may be duly ſituated, and not thruſt another out of its right place, which it will, if it be not level, but falling down towards a valley or riſing up a hill, or perhaps both; a line over ſuch ground will be falſe as to the plot, and therefore muſt be reduced to a level, and ſo taken off the ſcale, and protracted in the following manner:




Suppoſe ABC to be part of a hill falling within your plot, your boundary line going crooked from A to B, following the ſurface of the ground. To find the horizontal line, equal to AC, cauſe ſome perſon to ſtand at the point A, the foot of the hill, and to hold up the end of the chain to a convenient height, and gently aſcending the hill, you muſt draw it level, and make a mark where it toucheth the hill, obſerving the number of links between your aſſiſtant's hand, and that place where he muſt take his ſecond ſtanding, and hold it up as before, and ſo draw it out level again, 'till it touch the place where he muſt take his third ſtanding, noting the links as before; and ſo proceed, 'till at laſt, from his fifth ſtanding, you draw the chain level to the higheſt point within your plot, viz. the point B. And as the dotted lines of this figure were evidently equal to the line AC, ſo are the links noted down at every ſtation, when ſum-

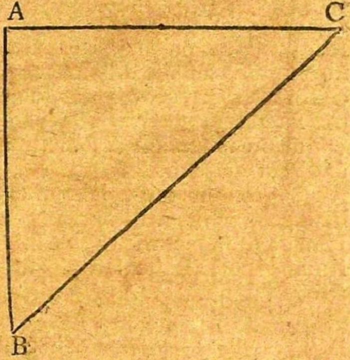
med

med up, equal to the horizontal line of that part of the hill.

In the very ſame manner, only inverting the order, you may find the horizontal lines going down hill, where that is moſt convenient; and if there be both aſcents and deſcents in one line betwixt two angles, the horizontal lines of both muſt be found and joined together without protraction.

All that has been ſaid concerning inequalities of riſing or falling ground, is to be underſtood when they are conſiderable, and a very exact plot required; for ſmall ones, eſpecially when much exactneſs is not expected, are not to be attended to.

*Concerning taking inacceſſible diſtances by the plain table, and acceſſible altitudes by the protractor.*




In order to anſwer queſtions of this kind, it will be neceſſary to meaſure the diſtance between two ſtations, properly choſen on the banks of the river, from whence ſome object on the oppoſite bank may be ſeen; then take the angles at each ſtation, and meaſure very accurately the diſtance between them; theſe obſervations being plotted, the diſtance may be eaſily meaſured.

EXAMPLE.

Let AC be the unknown breadth of a river, over which the General, that he may inform himſelf how many boats and planks are neceſſary to be brought, commands you to tell him the true diſtance from A, where he is at preſent to C, a little boat-houſe on the other ſide the water.

To ſatisfy his demand, plant your table covered with paper at A, cauſing a mark to be ſet up at B, at a good diſtance from you, along the bank of the river, the further the better, if the diſtance does not hinder the ſight; then having choſen a point to repreſent A, and laid the fiducial edge upon it, direct your ſights towards C and B, and ſtrike lines towards them, which done ſet up a mark at A, and from thence meaſure to B, ſix chains, thirty-two links, and plant your inſtrument at B, laying the fiducial edge to the line A B, and turning about the head of the inſtrument upon the ſtaff, till through the ſights you ſee the mark at A, and then ſcrew it faſt.

In the laſt place take ſix chains, thirty-two links off your ſcale, and ſet it on the line AB, from A to B, and laying the fiducial edge to the point B, from thence direct the ſights to C, and draw the line BC, meeting or cutting the line AC in C: ſo ſhall the ſpace AC meaſured on the ſcales, viz. eight chains, twenty-nine links, be the diſtance required; and becauſe the chain is twenty-two yards long, if you multiply 8.29 by 22, the product is 182 yards, and $\frac{38}{100}$ of a yard, which by reduction is ſomething more than 13 inches and $\frac{3}{5}$ of an inch.

How to take the height of a tree, tower, or ſteeple by a protractor, without any arithmetical operation. Hang a plummet with a fine ſilk thread to the center of it, and hold it ſtedfaſtly with that end to your eye where the numbers begin, then look ſtreight along the diameter, ſtill removing backward and forward as there is occaſion, till you ſee the top of the tree, tower, or ſteeple, and the thread at the ſame time fall upon forty-five degrees, ſo ſhall the diſtance from your eye to the tree, tower, or ſteeple, meaſured in an horizontal or level line, together with the height of your eye above the bottom of it, be equal to the height of the object.

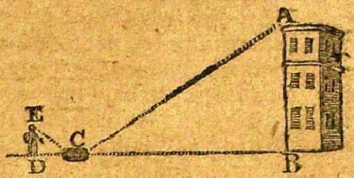
If either for convenience of ſight, or any other reaſon you think proper to place the other end of the diameter to your eye, then the thread for the above trial muſt fall upon 135 deg. inſtead of 45.

FIG.

## FIG. VI.

*To measure an accessible height by means of a plain Mirror.*

Let AB be the height to be measured; let the mirror be placed at C, in the horizontal plane BD, at a known distance BC; let the observer go back to D, till he see the image of the summit in the mirror, at a certain point of it, which he must diligently mark; and let DE be the height of the observator's eye. The triangles ABC and EDC are æquiangular; for the angles at D and B are right angles; and ACB, ECD, are equal, being the angles of incidence and reflection of the ray AC, as is demonstrated in optics; wherefore the remaining angles at A, and E, are also equal: Therefore, by 4th, 6. Eucl. it will be, as CD to DE, so CB to BA; that is, as the distance of the observator from the point of the mirror in the right line betwixt the observator and the height, is to the height of the observator's eye, so is the distance of the tower from that point of the mirror, to the height of the tower sought; which therefore will be found by the rule of three.




Note 1. The observator will be more exact, if, at the point D, a staff be placed in the ground perpendicularly, over the top of which the observator may see a point of the glass exactly in a line betwixt him and the tower.

Note 2. In place of a mirror may be used the surface of water contained in a vessel, which naturally becomes parallel to the horizon.

## FIG. VII.

*To measure an accessible height* AB *by means of two staffs.*

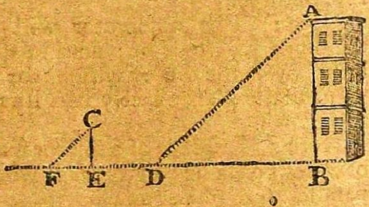
Let there be placed perpendicularly in the ground a longer staff DE, likewise a shorter one FG, so as the

observator may see A, the top of the height to be measured over the ends D, F, of the two staffs; let FH and DC, parallel to the horizon, meet DE and AB in H and C; then the triangles FHD, DCA, shall be æquiangular; for the angles at C and H are right ones; likewise the angle A, is equal to the angle FDH, by 29. 1. *Eucl.*; wherefore the remaining angles DFH, and ADC, are also equal: wherefore by 4. 6. *Eucl.* as FH, the distance of the staffs, to HD, the excess of the longer staff above the shorter; so is DC, the distance of the longer staff from the tower, to CA, the excess of the height of the tower above the longer staff. And thence CA will be found by the rule of three.

To which if the length DE be added, you will have the whole height of the tower BA.

## *SCHOLIUM.*

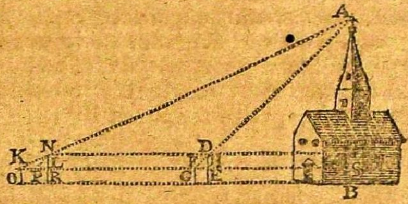
Many other methods may be occasionally contrived for measuring an accessible height. For example, from the given length of the shadow BD, I find out the height AB, thus: Let there be erected a staff CE perpendicularly, producing the shadow EF: The triangles ABD, CEF, are æquiangular; for the angles at B, and E, are right; and the angles ADB, CFE are equal each being equal to the angle of the sun's elevation above the horizon: therefore, by 4th, 6. *Eucl.* as EF the shadow of the staff, to EC the staff itself, so ED the shadow of the tower, to BA the height of the tower. Though the plane on which the shadow of the tower falls be not parallel to the horizon, if the staff be erected in the same plane, the rule will be the same.




To

*To meaſure an inacceſſible height by means of two ſtaffs.*

Hitherto we have ſuppoſed the height to be acceſſible, or that we can come at the lower end of it; now if, becauſe of ſome impediment, we cannot get to a tower, or if the point whoſe height is to be found out, be the ſummit of a hill, ſo that the perpendicular be hid within the hill; if, I ſay, for want of better inſtruments, ſuch an inacceſſible height is to be meaſured by means of two ſtaffs, let the firſt obſervation be made with the ſtaffs DE and FG, as in prop. I. then the obſervator is to go off in a direct line from the height and firſt ſtation, till he come to the ſecond ſtation; where he is to place the longer ſtaff perpendicularly at RN, and the ſhorter ſtaff at KO, ſo that the ſummit A may be ſeen along their tops; that is, ſo that the points KNA may be in the




ſame right line. Thro' the point N let there be drawn the right line NP parallel to FA: wherefore in the triangles KNP, KAF, the angles KNP, KAF are equal, by the 29. 1. *Eucl.* alſo the angle AKF is common to both; conſequently the remaining angle KPN, is equal to the remaining angle KFA. And therefore, by 4th, 6. *Eucl.* PN : FA : : KP : KF. But the triangles PNL, FAS are ſimilar; therefore, by 4th, 6. *Eucl.* PN : FA : : NL : SA. Therefore, by the 11. 5. *Eucl.* KP : KF : : NL : SA. Thence, alternately, it will be, as KP (the exceſs of the greater diſtance of the ſhort ſtaff from the long one above its leſſer diſtance from it) to NL, the exceſs of the longer ſtaff, above the ſhorter; ſo KF, the diſtance of the two ſtations of the ſhorter ſtaff, to SA, the exceſs of the height ſought above the height of the ſhorter ſtaff. Wherefore SA will be found by the rule of three. To which let the height of the ſhorter ſtaff be added, and the ſum will give the whole inacceſſible height BA.

Note

Note 1. In the ſame manner may an inacceſſible height be found by a geometrical ſquare, or by a plain ſpeculum. But we ſhall leave the rules to be found out by the ſtudent for his own exerciſe.

Note 2. That by the height of the ſtaff we underſtand its height above the ground in which it is fixed.

Note 3. Hence depends the method of uſing other inſtruments invented by Geometricians; for example, of the Geometrical croſs: and if all things be juſtly weighed, a like rule will ſerve for it as here. But we incline to touch only upon what is moſt material.

## FIG. IX.

*To meaſure the diſtance* AB, *to one of whoſe extremities we have acceſs, by the help of four ſtaffs.*

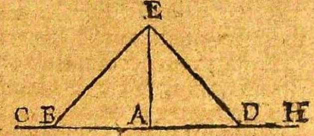
Let there be a ſtaff fixed at the point A; then going back at ſome ſenſible diſtance in the ſame right line, let another be fixed in C, ſo as that both the points A and B be covered and hid by the ſtaff C; likewiſe going off in a perpendicular from the right line CB, at the point A, (the method of doing which ſhall be ſhown in the following *Scholium*), let there be placed another ſtaff at H; and in the rightline CKG (perpendicular to the ſame CB, at the point C), and at the point of it K, ſuch that the points K, H, and B, may be in the ſame right line, let there be fixed a fourth ſtaff.

Let there be drawn, or let there be ſuppoſed to be drawn, a right line HG parallel to CA. The triangles KHG, HAB will be æquiangular; for the angles HAB, KGH are right angles. Alſo, by 29th, 1. *Eucl.* the angles ABH, KHG are equal; wherefore, by the 4th, 6. *Eucl.* as KG (the exceſs of CK above AH) to GH, or to CA, the diſtance betwixt the firſt and ſecond ſtaff; ſo is AH, the diſtance betwixt the firſt and third ſtaff, to AB the diſtance ſought.

SCHO-

### SCHOLIUM.

To draw on a plane right line AE perpendicular to CH, from a given point A; take the right lines AB, AD, on each ſide equal; and in the points B and D, let there be fixed ſtakes, to which let there be tied two equal ropes BE, DE, (or one having a mark in the middle, and holding in your hand their extremities joined, (or the mark in the middle, if it be but one), draw out the ropes on the ground; and then, where the two ropes meet, or at the mark, when by it the rope is fully ſtretched, let there be placed a third ſtake at E; the right line AE will be perpendicular to CH in the point A, by 11th, 1. *Eucl.* In a manner not unlike to this may any problems that are reſolved by the ſquare and compaſſes, be done by ropes and a cord turned round as a radius.




## CHAP. II.

### *Of meaſuring* Bricklayer's Work.

*Diſcipulus.* IS there any difference between meaſuring bricklayer's work, and meaſuring ſuperficies in general?

*Philo.* There is no difference in finding the content; but there is a very material difference in taking the dimenſions, and finding the true value of any piece of work; becauſe walls are of different thickneſs; ſometimes ornamented, ſometimes not; it will therefore be neceſſary to explain theſe particulars; for otherwiſe it will be impoſſible to meaſure truly the ſeveral particulars of bricklayer's work.

*Diſcip.* How are the dimenſions taken?

*Philo.* The dimenſions in brick-work, are eſtimated by the rod, conſiſting of 16 feet and a half, therefore the reſult is in ſquare rods of $272\frac{1}{4}$ ſquare feet each.

Hence, in practice, they divide the area found in ſquare feet by 272, and the quotient will be the rods required.

Bricklayers always compute or value their work at the rate of a brick and an half thick; and if the thickneſs of the wall happen to be more or leſs than that ſtandard, it muſt be reduced to the above thickneſs, in the following manner.

Multiply the area of the wall, by the number of half bricks, contained in the thickneſs of the wall; divide the product by 3, and the quotient is the area required.

Brick walls are ſometimes wrought part of the way two inches thicker than the reſt of the work; theſe two inches ſerving as a water-table to the wall, which is commonly ſet off about two feet above the ground; the brick-work may therefore be meaſured at the ſame thickneſs, which is above the water-table; and then the two-inch-work may be meaſured, and afterwards reduced to that of a brick and a half.

*Diſcip.* I now ſufficiently underſtand why there is a neceſſity for dividing general menſuration of ſuperficies into different kinds: but you alſo mentioned ſomething of ornamental work; I ſhould be glad to hear you explain what you meant by that expreſſion?

*Philo.* By ornamental mark is meant the arches over doors, windows, &c. Facias, Architraves, rubbed Returns, Friezes, Cornices, Ruſtices, &c. Piers, Columns, Pilaſters, &c. which are valued by the ſquare foot. But the bricklayers in general underſtand, by ornamental work, thoſe particular parts in a building, which are ground, gauged, or poliſhed. Others call carved mouldings, &c. by the name of ornamental work; but this is generally undertaken at a certain price per foot, running meaſure.

*Diſcip.* I am greatly obliged to you, Sir, for your kind intelligence; but ſhould be glad to know how the dimenſions of ſuch works are taken.

*Philo.* When the content is to be found in ſquare meaſure, the length is meaſured where it is greateſt, and the height or breadth is meaſured by extending the line from the upper part of the ſurface, to what workmen call the Soffie, or under part of it, includ-

ing

ing in this the bottom home to the back part of the bricks. The ſame method is obſerved in meaſuring Cornices, and Facias, &c. They are girt over all their ſurfaces, including both the upper and under returns, no part being omitted.

It is alſo neceſſary to obſerve, that when two walls join in an angle, they meaſure the length of one on the outſide, and that of the other on the inſide; the ſum of theſe two menſurations, being conſidered as the true length of the whole wall.

In all buildings of any height, the thickneſs of the walls decreaſes as they riſe; and this decreaſe conſiſts generally of half a brick. The thickneſs is ſet off on the inſide, and generally in a place where the floor will be laid; a contrivance by which the ſet-off is concealed. Theſe different thickneſſes therefore muſt be meaſured ſeparately, their areas found, and then added together; for that ſum gives the true content of the wall.

If a chimney ſtand by itſelf, without any party-wall being joined to it, they meaſure it in the following manner: they girt it about for the length, and reckon the height of the ſtory for the breadth: The thickneſs muſt be the ſame with that of the jaumbs, provided the chimney be wrought upright from the mantle-tree to the cieling; not deducting any thing for the vacancy, between the mantle-tree and the hearth, becauſe of the gathering of the breaſt and wings to make room for the hearth in the next ſtory.

But when the chimney-back is a party-wall, and the wall meaſured by itſelf; they take the depth of the two jaumbs, and the length of the breaſt for the length; the height of the ſtory for the breadth; and the thickneſs of the jaumbs for the proper thickneſs. With theſe dimenſions the content of the chimney is eaſily found.

Chimney-ſhafts, or thoſe parts which appear above the roof, are meaſured by girting them with a line about the leaſt part for the length; and taking the height for the breadth, if they are four inches thickneſs and one brick; but if they are nine inches, they

reckon the thickneſs at a brick and a half, in conſideration of their plaiſtering, or as they call it, pargeting, the inſide of the ſhaft, and the trouble of raiſing the ſcaffold. In ſome places, indeed, it is cuſtomary to allow double meaſure for chimnies.

When the chimney ſtands in one of the angles or corners of the room, they multiply half the product of the height into the breadth of the breaſt or front, and reckon the thickneſs by that of the jaumbs.

But you muſt always remember in meaſuring brick-work, that you muſt deduct the ſpaces left for doors, windows, &c.

*Diſcip.* I am highly obliged to you for this explanation, and perceive that the difficulty conſiſts in taking the dimenſions properly, and reducing the whole to a brick and a half thick; that is, by multiplying the area by the number of half bricks, and dividing the product by 3.

*Philo.* You are extremely right, Diſcipulus; and in order to ſhorten the latter part of the work, I mean, to ſave the trouble of multiplying the area by the number of half bricks in the wall, &c. and dividing that product by 3, I have calculated the following TABLE, which will ſave a great deal of trouble, and remove the danger of committing miſtakes.

# A
# TABLE

For reducing Brick-work of any thickneſs to the ſtandard thickneſs of a brick and a half.

| | | ½ Brick. | | | 1 Brick. | | | 1½ Brick. | | | 2 Bricks. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | R. | Q. | F. | R. | Q. | F. | R: | Q. | F. | R. | Q. | F. |
| 1 Quarter. | | 0 | 0 | 22 | 0 | 0 | 45 | 0 | 1 | 0 | 0 | 1 | 22 |
| 2 Quarters. | | 0 | 0 | 45 | 0 | 1 | 22 | 0 | 2 | 0 | 0 | 2 | 45 |
| 3 Quarters. | | 0 | 2 | 0 | 0 | 2 | 0 | 0 | 3 | 0 | 1 | 0 | 0 |
| Rods contained in the ſuperficial meaſure of the wall. | 1 | 0 | 1 | 22 | 0 | 2 | 45 | 1 | 0 | 0 | 1 | 0 | 22 |
| | 2 | 0 | 2 | 45 | 1 | 1 | 22 | 2 | 0 | 0 | 2 | 1 | 45 |
| | 3 | 1 | 0 | 0 | 2 | 0 | 0 | 3 | 0 | 0 | 4 | 0 | 0 |
| | 4 | 1 | 1 | 22 | 2 | 2 | 45 | 4 | 0 | 0 | 5 | 1 | 22 |
| | 5 | 1 | 2 | 45 | 3 | 1 | 22 | 5 | 0 | 0 | 6 | 2 | 45 |
| | 6 | 2 | 0 | 0 | 4 | 0 | 0 | 6 | 0 | 0 | 8 | 0 | 0 |
| | 7 | 2 | 1 | 22 | 4 | 2 | 45 | 7 | 0 | 0 | 9 | 1 | 22 |
| | 8 | 2 | 2 | 45 | 5 | 1 | 22 | 8 | 0 | 0 | 10 | 2 | 45 |
| | 9 | 3 | 0 | 0 | 6 | 0 | 0 | 9 | 0 | 0 | 12 | 0 | 0 |
| | 10 | 3 | 1 | 22 | 6 | 2 | 45 | 10 | 0 | 0 | 13 | 1 | 22 |
| | 11 | 3 | 2 | 45 | 7 | 1 | 22 | 11 | 0 | 0 | 14 | 2 | 45 |
| | 12 | 4 | 0 | 0 | 8 | 0 | 0 | 12 | 0 | 0 | 16 | 0 | 0 |
| | 13 | 4 | 1 | 22 | 8 | 2 | 45 | 13 | 0 | 0 | 17 | 1 | 22 |
| | 14 | 4 | 2 | 45 | 9 | 1 | 22 | 14 | 0 | 0 | 18 | 2 | 45 |
| | 15 | 5 | 0 | 0 | 10 | 0 | 0 | 15 | 0 | 0 | 20 | 0 | 0 |
| | 16 | 5 | 1 | 22 | 10 | 2 | 45 | 16 | 0 | 0 | 21 | 1 | 22 |
| | 17 | 5 | 2 | 45 | 11 | 1 | 22 | 17 | 0 | 0 | 22 | 2 | 45 |
| | 18 | 6 | 0 | 0 | 12 | 0 | 0 | 18 | 0 | 0 | 24 | 0 | 0 |
| | 19 | 6 | 1 | 22 | 12 | 2 | 45 | 19 | 0 | 0 | 25 | 1 | 22 |
| | 20 | 6 | 2 | 45 | 13 | 1 | 22 | 20 | 0 | 0 | 27 | 2 | 45 |
| | 21 | 7 | 0 | 0 | 14 | 0 | 0 | 21 | 0 | 0 | 28 | 0 | 0 |

# A
# TABLE

For reducing Brick-work of any thickneſs to the ſtandard thickneſs of one brick and a half.

| | | 2½ Bricks | | | 3 Bricks. | | | 3½ Brieks | | | 4 Bricks. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | R. | Q. | F. | R. | Q. | F. | R. | Q. | F. | R. | Q. | F. |
| 1 Quarter. | | 0 | 1 | 45 | 0 | 2 | 0 | 0 | 2 | 22 | 0 | 2 | 45 |
| 2 Quarters. | | 0 | 3 | 22 | 1 | 0 | 0 | 1 | 0 | 45 | 1 | 1 | 22 |
| 3 Quarters. | | 1 | 1 | 0 | 1 | 2 | 0 | 1 | 3 | 0 | 2 | 0 | 0 |
| Rods contained in the ſuperficial meaſure of the wall. | 1 | 1 | 2 | 45 | 2 | 0 | 0 | 2 | 1 | 22 | 2 | 2 | 45 |
| | 2 | 3 | 1 | 22 | 4 | 0 | 0 | 4 | 2 | 45 | 5 | 1 | 22 |
| | 3 | 5 | 0 | 0 | 6 | 0 | 0 | 7 | 0 | 0 | 8 | 0 | 0 |
| | 4 | 6 | 2 | 45 | 8 | 0 | 0 | 9 | 1 | 22 | 10 | 2 | 45 |
| | 5 | 8 | 1 | 22 | 10 | 0 | 0 | 11 | 2 | 45 | 12 | 1 | 22 |
| | 6 | 10 | 0 | 0 | 11 | 0 | 0 | 14 | 0 | 0 | 16 | 0 | 0 |
| | 7 | 11 | 2 | 45 | 14 | 0 | 0 | 16 | 1 | 22 | 18 | 2 | 45 |
| | 8 | 13 | 1 | 22 | 16 | 0 | 0 | 18 | 2 | 45 | 21 | 1 | 22 |
| | 9 | 15 | 0 | 0 | 18 | 0 | 0 | 21 | 0 | 0 | 24 | 0 | 0 |
| | 10 | 16 | 2 | 45 | 20 | 0 | 0 | 23 | 1 | 22 | 26 | 2 | 45 |
| | 11 | 18 | 1 | 22 | 22 | 0 | 0 | 25 | 2 | 45 | 29 | 1 | 22 |
| | 12 | 20 | 0 | 0 | 24 | 0 | 0 | 28 | 0 | 0 | 32 | 0 | 0 |
| | 13 | 21 | 2 | 45 | 26 | 0 | 0 | 30 | 1 | 22 | 34 | 2 | 45 |
| | 14 | 23 | 1 | 22 | 28 | 0 | 0 | 32 | 2 | 45 | 37 | 1 | 22 |
| | 15 | 25 | 0 | 0 | 30 | 0 | 0 | 35 | 0 | 0 | 40 | 0 | 0 |
| | 16 | 26 | 2 | 45 | 32 | 0 | 0 | 37 | 1 | 22 | 42 | 2 | 45 |
| | 17 | 28 | 1 | 22 | 34 | 0 | 0 | 39 | 2 | 45 | 45 | 1 | 22 |
| | 18 | 30 | 0 | 0 | 36 | 0 | 0 | 42 | 0 | 0 | 48 | 0 | 0 |
| | 19 | 31 | 2 | 45 | 38 | 0 | 0 | 44 | 1 | 22 | 50 | 2 | 45 |
| | 20 | 33 | 1 | 22 | 40 | 0 | 0 | 46 | 2 | 45 | 53 | 1 | 22 |
| | 21 | 35 | 0 | 0 | 42 | 0 | 0 | 49 | 0 | 0 | 56 | 0 | 0 |

A TABLE

# A TABLE

For reducing Brick-work of any thickneſs to the ſtandard thickneſs of a brick and a half.

| | 4½ Bricks | | | 5 Bricks. | | | 5½ Bricks | | |
|---|---|---|---|---|---|---|---|---|---|
| | R. | Q. | F. | R. | Q. | F. | R. | Q. | F. |
| 1 Quarter. | 0 | 2 | 0 | 0 | 5 | 22 | 0 | 3 | 45 |
| 2 Quarters. | 1 | 3 | 0 | 1 | 2 | 45 | 1 | 3 | 22 |
| 3 Quarters. | 2 | 1 | 0 | 2 | 2 | 0 | 2 | 3 | 0 |
| Rods contained in the ſuperficial meaſure of the wall. 1 | 3 | 0 | 0 | 3 | 1 | 22 | 3 | 2 | 45 |
| 2 | 6 | 0 | 0 | 6 | 2 | 45 | 7 | 1 | 22 |
| 3 | 9 | 0 | 0 | 10 | 0 | 0 | 11 | 0 | 0 |
| 4 | 12 | 0 | 0 | 13 | 1 | 22 | 14 | 2 | 45 |
| 5 | 15 | 0 | 0 | 16 | 2 | 45 | 18 | 1 | 22 |
| 6 | 18 | 0 | 0 | 20 | 0 | 0 | 22 | 0 | 3 |
| 7 | 21 | 0 | 0 | 23 | 1 | 22 | 25 | 2 | 45 |
| 8 | 24 | 0 | 0 | 26 | 2 | 45 | 29 | 1 | 22 |
| 9 | 27 | 0 | 0 | 30 | 0 | 0 | 33 | 0 | 0 |
| 10 | 30 | 0 | 0 | 33 | 1 | 22 | 36 | 2 | 45 |
| 11 | 33 | 0 | 0 | 36 | 2 | 45 | 40 | 1 | 22 |
| 12 | 36 | 0 | 0 | 40 | 6 | 0 | 44 | 0 | 0 |
| 13 | 39 | 0 | 0 | 43 | 1 | 22 | 47 | 2 | 45 |
| 14 | 42 | 0 | 0 | 46 | 2 | 45 | 51 | 1 | 22 |
| 15 | 45 | 0 | 0 | 50 | 0 | 0 | 55 | 0 | 0 |
| 16 | 48 | 0 | 0 | 53 | 1 | 22 | 58 | 2 | 45 |
| 17 | 51 | 0 | 0 | 56 | 2 | 45 | 62 | 1 | 22 |
| 18 | 54 | 0 | 0 | 60 | 0 | 0 | 66 | 0 | 0 |
| 19 | 57 | 0 | 0 | 63 | 1 | 22 | 69 | 2 | 45 |
| 20 | 60 | 0 | 0 | 66 | 2 | 45 | 73 | 1 | 22 |
| 21 | 63 | 0 | 0 | 70 | 0 | 0 | 77 | 0 | 0 |

EXPLA-

## EXPLANATION of the foregoing TABLE.

At the head of the table is ſet the thickneſs of the wall, in bricks and half bricks, from half a brick in thickneſs to ſix bricks.

In the firſt column is placed the number of the rods contained in any wall, from one quarter of a rod to twenty-one rods; and againſt thoſe numbers, in the other columns, ſtands the quantity of brick-work, in rods, quarters, and feet which a wall contains of any of theſe thickneſſes, at the head of the table.

### EXAMPLE I.

If a wall upon the ſuperficies contains twelve rods, and it be three bricks and a half in thickneſs, how many rods does the wall contain of the ſtandard thickneſs of a brick and a half?

Find twelve rods in the firſt, and under three bricks, $\frac{1}{2}$, at the head of the table, in the angle of meeting, you will find 28 rods; which is the true quantity required at the ſtandard thickneſs of a brick and half.

### EXAMPLE II.

If a wall be four bricks and a half thick, and contains upon the ſuperficies nineteen rods, how many rods of brick-work are contained in that wall, at the ſtandard thickneſs?

Look for nineteen rods in the firſt column, and under 4 bricks $\frac{1}{2}$, you will find fifty-ſeven rods, the content required.

The ſame may be obſerved as to any other wall.

*Diſcip.* I think your directions ſo very plain, that I thoroughly underſtand the whole, and therefore ſhall not give you the trouble of anſwering any queſtions.

*Philo.* It gives me great pleaſure to hear you underſtand my inſtructions ſo readily; indeed I have endeavoured to render them as plain and evident, as the nature

ture of the subject will admit. But before I conclude this chapter of bricklayer's work, it will be necessary to explain the method

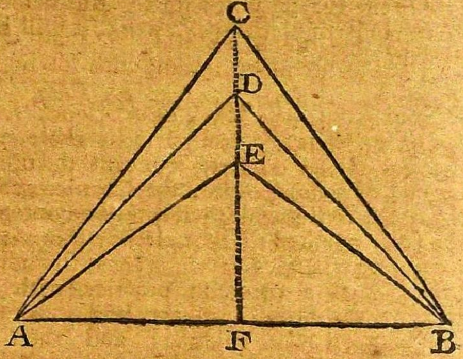
*Of measuring gable-ends.*

*Discip.* What do you mean by gable-ends?

*Philo.* By a gable-end is meant the upright triangular end of a house, from the Cornice or Eaves, to the top of the roof; and therefore the content is found by the same method as that already given in page 17, for measuring triangles.

But it is a common practice with workmen to measure the breadth of the house, which constitutes the base of the triangle; and then measure the length of the rafter, adding to it half its length; and the half of that sum will be the length of the perpendicular required. But this rule is true only when the roof is what they call *a true pitch*; that is, when the rafters are three quarters of the breadth of the building.

I would therefore always advise you to measure the length of the perpendicular; or, if that cannot be easily done, to square both the length of the rafter, and half the length of the base; subtract the lesser from the greater, and extract the square root out of the remainder, which will be the true length of the perpendicular required.




As the measuring of gable-ends is of some consequence in finding the contents of bricklayers work, and is often mistaken by workmen, the diagram before you will explain the true method of proceeding, and point out the errors that often arise by following the common rule, with regard to the length of the perpendicular.

The triangle ADB, represents a gable-end of a true pitched roof, that is, when the length of the rafter AD=DB, is three fourths of the length of the base AB. In this case the length of the perpendicular DF, will be truly found by the common method: But if the roof be above true pitch; that is, if the rafter AC be more than three fourths of the base AB, the length of the perpendicular CF, cannot be found sufficiently near the truth by the common rule. For C F is considerably greater than DF; and consequently the whole space between ACB and ADB will be lost to the workman. On the contrary, if the roof be below the true pitch; that is, if the rafter AE=EB be less than three fourths of the base AB, the perpendicular EF will be shorter than DF; and consequently the workman will be paid for the space between ADB, and AEB, above what he has really done. I would therefore advise you never to trust to the workmen's common rule, 'till you are assured by actual mensuration, that the roof is truly pitched; because you are then sure of being right, and because the true method is attended with very little trouble, more than the common rule.

These

Theſe particulars being well conſidered, you will find no difficulty in meaſuring any kind of works performed by the bricklayer: I ſhall therefore finiſh this ſpecies of menſuration, with ſolving the following queſtion.

What will the bricklayer's work for erecting the walls of a houſe, at 5l. 10s. the rod, amount to, ſuppoſing the walls to be of the following dimenſions: The ſide-walls 28 feet 6 inches in length, the height of the roof from the ground 55 feet 6 inches, and the gable to riſe 10 feet 6 inches, the roof being a true pitch: The ſide-walls from the bottom to 20 feet at 2 bricks and a half thick: 20 feet more at 2 bricks: 15 feet 6 inches more at 1 brick and a half thick; and the gable-end 1 brick thick?

## SOLUTION.

### FIRST DIMENSIONS.

```
    28.5   The length.
      20
  ------
   570.0
       5   Half bricks thick.
  ------
3|2850|950 Area in feet.
  ...
   27
  ----
   15
   15
  ----
   ..0
```

## SECOND DIMENSIONS.

```
    28.5
      20
  ------
   570.0
       4
  ------
 3|228.0|760   Area in feet.
   21
  ------
   · 18
     18
  ------
   · ·0
```

## THIRD DIMENSIONS.

```
     28.5
     15.5
   ------
     1425
    1425
    285
   ------
   441.75   Area in feet.
```

## FOURTH DIMENSIONS, or GABLE-END.

```
     28.5
      5.3   Half perpendicular of the gable.
   ------
     855
   1425
   ------
   151.05
        2
   ------
 3|302.10|100.7   Area in feet.
   3 · ·
   ------
   ·021
     21
   ------
     · ·
```

```
  950     First dimensions.
  760     Second
  441.75  Third
  100.7
-------
 2252.45
```

Then as 272 feet (a square rod)

```
                     F.          L.
is to 5.5 : : 2252.45 : 45.545
                  5.5
              -------
               126225
              1126225
         ------------
   272)12388.475            (45.545
        1088 . .                 20
        -----                ------
        .1508                10.900 .
         1360                    12
        -----                ------
        .1484                10.800
         1360                     4
        -----                ------
        .1247                 3.200
         1088
        -----
        .1595
         1360
        -----
         .235
```

Answ. 45*l.* 10*s.* 10*d.* ¾

## CHAP. III.

### *Of measuring* CARPENTERS WORK.

*Discip.* What methods are made use of by carpenters in measuring their works?

*Philo.* Carpenters, in measuring their works, have recourse to different methods; because the different parts of their employment render different methods of mensuration necessary. But at the same time you must remember, that these methods vary only in taking the dimensions; for when these are truly found, the contents will be had, from the common method of measuring geometrical figures.

*Discip.*

*Discip.* This explanation has removed several difficulties which occurred to my mind. I was persuaded, that the areas were found by the same methods that are used for finding them in similar figures; and therefore required very little explanation; as I hope, I have sufficiently retained all the rules you have been kind enough to lay down for that purpose. But I shall now listen very attentively to your explanation of the methods taken by these practical artists.

*Philo.* Carpenters works, which are measurable, are flooring, partitioning, and roofing, all which are measured by the square of ten feet long, and ten broad, so that one square contains an hundred square feet.

## *Of* FLOORING.

The dimensions of floors are taken within the walls; but those of joists and girders are measured by the length of the room, and also by the distance they are let into the walls.

## EXAMPLE. I.

If a floor be 57 feet 3 inches long, and 28 feet 6 inches broad, how many squares of flooring are there in that room? To know this, multiply 57 feet 3 inches, by 28 feet 6 inches, and the product will be 1631; divide this by 100, which is done by cutting off two figures towards the right hand with a dash of the pen, and the remaining figures are the quotient, and the figures cut off are feet. Thus if you divide 1631 by 100, that is cut off 31 from the right hand, the quotient will be 16 squares, and the 31 cut off be 31 feet.

Operation

*Operation by decimals, and also by feet and inches.*

| Decimals | F. I. |
|---|---|
| 57.25 | 57.3 |
| 28. 5 | 28.6 |
| 28625 | 456 |
| 45800 | 114 |
| 11450 | 28.7.6 |
| 16\|31.625 | 7.0.0 |
| | 16\|31.7.6 |

Answ. 16 squares, 31 feet.

## EXAMPLE II.

If a floor be 53 feet 6 inches long, and 47 feet 9 inches broad; how many squares are contained in that floor?

## OPERATION.

| Decimals | F. I. |
|---|---|
| 47.75 | 53.6 |
| 53. 5 | 47.9 |
| 23875 | 371 |
| 14325 | 212 |
| 23875 | 26.9 |
| 25\|54.625 | 13.4.6 |
| | 23.10.6 |
| | 25\|55 . 0.0 |

Answ. 25 squares, 55 feet

## *Of* PARTITIONING.

The dimensions of partitions are taken in the same manner as those of floors, and therefore require no farther explanation.

## EXAMPLE I.

If a partition between rooms be in length 82 feet 6 inches, and in height 12 feet 5 inches; how many squares are contained therein?

*Operation by decimals, and alſo by feet and inches.*

```
   12.25                          F. I.
   82. 5                          82.6
  ------                          12.3
   6125                          ------
  2450                           990.0
 9800                             20.7.6
--------                         --------
10|10.625                        10 10.7.6
```

Anſw. 10 ſquares, 10 feet.

## EXAMPLE II.

If a partition between rooms, be in length 19 feet 9 inches, and in breadth 11 feet 3 inches; how many ſquares are contained therein?

*Operation by decimals.*

```
   19.75
   11.25
  ------
    9875
   3950
  1975
 1975
--------
2|22.1875
```

Anſwer, 2 ſquares, 22 feet.

## OF ROOFING.

It is a rule among workmen, that the flat of any houſe, and half the flat thereof taken within the walls, is equal to the meaſure of the roof of the ſame houſe; but this is when the roof is true pitched; for if the roof be more flat or ſteep than the true pitch, it will meaſure more or leſs accordingly; multiply therefore the length and breadth together, to that product add half the ſum, and the total being divided by 100, will give the anſwer in ſquares.

EXAMPLE.

EXAMPLE.

If a houſe within the walls be 44 feet 6 inches long, and 18 feet 3 inches broad, how many ſquares of roofing will cover that houſe?

| | |
|---|---|
| | F. I. |
| | 44.6 |
| 18.25 | 18.3 |
| 41.5 | 352 |
| 9125 | 44 |
| 7300 | 11.1.6 |
| 7300 | 9.0.0 |
| Flat 812.125 | Flat 812.1.6 |
| Half 406 | Half 406 |
| 12\|18 Anſw. 12 ſq. 18 feet. | 12\|18 |

In the meaſuring-work of carpenters, there are two things which ſhould be attended to:

Firſt, in the meaſuring of flooring, after the whole floor has been meaſured, there are to be deducted out of it the well-holes for the ſtairs and chimnies; and in partitioning, for the doors, windows, &c. except, by agreement, they are to be included.

Secondly. In meaſuring of roofing, ſeldom any deductions are made for the holes for the chimney ſhafts, or the vacancies for the lanthorn-lights and ſky-lights, as they are more trouble to the workman, than the ſtuff that would cover them is worth.

## CHAP. IV.

### OF MEASURING GLAZIERS-WORK.

Glaziers generally meaſure their work to a quarter of an inch, and never make any allowances for round or oval windows, but always meaſure them to the greateſt length; for there is more trouble in cutting the glaſs to thoſe ſhapes, than the value of the glaſs omitted.

When the dimenſions are thus accurately taken, the content in feet, inches, &c. is eaſily found by the me-

thods already laid down for meaſuring ſuperficies, and which need not be repeated here. But as theſe tradeſmen give particular appellations to the various dimenſions of quarries uſed in glazing windows; it will be neceſſary to add an explanation of theſe terms, and then ſhew how they are meaſured, and the content found in feet, inches, and parts.

The range, which is a perpendicular let fall from one of the obtuſe angles to the oppoſite ſide, when the length is the longeſt diagonal, from one acute angle to the other, and the breadth is the ſhorteſt diagonal, which is drawn between the two obtuſe angles; as for the ſides, and area of the quarry, theſe are well known to all.

Glaziers generally conſider twelve ſorts of quarries, from whence ariſe divers propoſitions of infinite uſe to Glaziers; as

Firſt. To find any of the dimenſions, as range, ſide, length, and breadth, of any ſort of quarries.

Secondly. To find the area of any ſort of quarries.

Thirdly. Having any of the dimenſions given, viz. range, ſide, breadth or length, to find the name or denomination of the ſize, viz. whether 8s. 10s. 12s. &c.

Fourthly. Having the area of a quarry given, to find of what ſort or ſize it is.

Fifthly. To find whether a window be glazed with thoſe, they call ſquare quarries or long ones; for it is to be obſerved, that there are ſix ſizes of ſquare quarries, and ſix ſizes of long quarries, which make twelve ſorts in all.

Glazier's work is meaſured by the foot ſquare; ſo that the length and breadth of a pane of glaſs in feet, being multiplied into each other, produceth the content.

It ſhould be remarked that Glaziers uſually take their dimenſions very accurately; and in multiplying feet inches, and parts, the inch is divided into 12 parts, as the foot is, and each part is alſo divided into 12.

## EXAMPLE I.

If a pane of glaſs be four feet eight inches, and three quarters long; and one foot, four inches, and one

one quarter broad, how many feet of glaſs does it contain?

The decimal of $\left\{\begin{array}{l}8 \text{ Inches}\frac{3}{4}\\4 \text{ Inches}\frac{1}{4}\end{array}\right\}$ is $\left\{\begin{array}{l}.729\\.354\end{array}\right.$

| F. | I. | P. | | |
|---|---|---|---|---|
| 4 | 8 | 9 | | |
| 1 | 4 | 3 | | |
| 4 | 8 | 9 | | |
| 1 | 6 | 11 | 0 | |
| | 1 | 2 | 2 | 3 |
| 6 | 4 | 8 | 2 | 3 |

```
  4.729
  1.354
  18916
 23645
14187
4729
6.403066
```

Facit 6 feet 4 inches.

EXAMPLE II.

If there be eight panes of glaſs, each four feet, ſeven inches, three quarters long, and one foot five inches one quarter broad, how many feet of glaſs are contained in the ſaid eight panes?

The decimal of $\left\{\begin{array}{l}7 \text{ Inches}\frac{3}{4}\\5 \text{ Inches}\frac{1}{4}\end{array}\right\}$ is $\left\{\begin{array}{l}646\\.437\end{array}\right.$

| F. | I. | P. | | |
|---|---|---|---|---|
| 4 | 7 | 9 | | |
| 1 | 5 | 3 | | |
| 4 | 7 | 9 | | |
| 1 | 11 | 2 | 9 | |
| | 1 | 1 | 11 | 3 |
| 6 : | 8 : | 1 : | 8 : | 3 |
| | | | | 8 |
| 53 : | 5 : | 1 : | 6 : | 0 |

```
    4.646
     1.45
    32522
   13938
  18584
 4646
 6.676302
        8
53.410416
```

Facit 53 feet 5 inches.

EXAMPLE III.

If there are ſixteen panes of glaſs, each four feet, five inches, and a half long, and one foot four inches and

 three

three quarters broad, how many feet of glaſs are contained in them ?

| F. I. P. | |
|---|---|
| 4 : 5 : 6 | 4.458 |
| 1 : 4 : 9 | 1.395 |
| 4 : 5 : 6 : | 22290 |
| 1 : 5 : 10 : | 40122 |
| 3 : 4 : 1 : 6 | 13374 |
| 6 : 2 : 8 : 1 : 6 | 4458 |
| 4 | 6.218910 |
| 24 : 10 : 8 : 6 : 0 | 4 |
| 4 | 24.875640 |
| 99 : 6 : 10 : 0 : 0 | 4 |
| | 99.502560 |

Facit 99 feet 6 inches.

It may be obſerved, that inſtead of multiplying by 16, I have multiplied by 4 twice, becauſe four times 4 is 16.

When windows have half rounds at the top, they are to be meaſured at their full height, as if they were ſquare. For the ſame reaſon, round or oval windows are meaſured at the full length and breadth of their diameters.

In like manner are crotchet-windows in ſtone-work meaſured by their full ſquares ; and for this plain reaſon, that the trouble of taking their dimenſions to work by, the waſte of glaſs in working, and the time ſpent in ſetting them up, is far more than the value of the glaſs.

## CHAP. V.

### OF MEASURING JOINERS-WORK.

Joiners meaſure their work by the yard ſquare ; but they take their dimenſions differently from others. In taking the height of a room, about which there is either cornice, ſwelling pannels, or mouldings, they begin at the top with a ſtring, and girt over all the mouldings, which makes the room meaſure much higher than it really

really is; but in meaſuring about the room, they take it only as it is upon the floor.

## Rule.

Multiply the compaſs by the height, and the product divided by 9 will give the anſwer.

## Example I.

If a room of wainſcot being girt downwards over the mouldings be 15 feet 9 inches high, and 126 feet 3 inches in compaſs, how many yards are contained in that room?

| F. | I. | |
|---|---|---|
| 126 | 3 | |
| 15 | 9 | |
| 630 | | |
| 126 | | |
| 63 | 1 | 6 |
| 31 | 6 | 9 |
| 3 | 9 | 0 |
| 9)1988 | 5 | 3 |

```
      126 : 25
       15 : 75
      63125
     88375
    63125
   12625
9)1988.4375
   220 : 8
```

Anſw. 220 Y : 8 F.

## Example II.

If a room of wainſcot be 16 feet 3 inches high, girt over the mouldings, and the compaſs of the room is 137 feet 6 inches, how many yards does it contain?

| | |
|---|---|
| F. I. | |
| 137 : 6 | 137.5 |
| 16 : 3 | 162.5 |
| 830 | 6875 |
| 137 | 2750 |
| 34 : 4 : 6 | 8250 |
| 9)2234 : 4 : 6 | 1375 |
| 248 : 2 : • | 9)2234.375 |
| | 248.2 |

Answ. 248 yards. 2 feet.

In meaſuring doors, window-ſhutters, and all ſuch work, as is wrought on both ſides, joiners are paid for work and half work; therefore in meaſuring all ſuch work, the content muſt firſt be found as before, and half that content muſt be added to it, which will give the content at work and half.

## EXAMPLE III.

If the window-ſhutters be 69 feet 9 inches broad, and 6 feet 3 inches high, what is their content at work and half?

| | |
|---|---|
| F. I. | |
| 69 : 9 | 69.75 |
| 6 : 3 | 6.25 |
| 418 : 6 | 34875 |
| 17 : 5 : 3 | 13950 |
| 435 : 11 : 3 | 41850 |
| 217 : 11 : 7 | 435.9375 |
| 9)653 : 10 : 10 | 217.9687 |
| 72 : 5 | 9)653.9062 |
| | 72.5 |

Answ. 72 yards, 5 feet.

CHAP.

# CHAP. VI.

## Of Measuring Painters-work.

The method of taking the dimensions here, is the same with that of joiners, by girting over the mouldings, &c. in taking the height. The casting up, after the dimensions have been taken and reduced into yards, is likewise the same with that of joiners work; but the painter never reckons work and half; but reckons his work once, twice or thrice coloured over.

It should however be observed, that window-lights, window-bars, casements, and such like things, are done at so much per piece.

### Example.

If a room be painted, whose height, being girt over the mouldings, is 16 feet 6 inches, and the compass of the room be 97 feet 9 inches, how many yards are there in that room?

```
     F.  I.                                   97.75
     97 : 9                                    16.5
     16 : 6                                  ------
   ---------                                  48875
     584                                     58650
     98                                      9775
      48 : 10 : 6                           ---------
   ---------------                         9)1612.875
  9)1612 : 10 : 6
```

Answer 179 yards, 1 foot.

# CHAP. VI.

## Of Measuring Plasterers-work.

Plasterers works are of two kinds:

First, Lathed and plastered work, which they call ceiling.

Secondly, Rendered work, which is of two sorts, viz. upon brick-walls or between quarters, and in the partitions between rooms; all which are measured by

by the yard ſquare, or ſquare of 3 feet, which is 9 feet.

## OF CIELING.

If a ceiling be 59 feet, 9 inches long, and 24 feet, 6 inches broad, how many yards are contained in that cieling?

### EXAMPLE.

| F. I. | |
|---|---|
| 59 : 9 | 59.75 |
| 24 : 6 | 24.5 |
| 236 | 29875 |
| 118 | 23900 |
| 18 : 00 : 0 | 11950 |
| 29 : 10 : 6 | 9)1463.875 |
| 9)1463 : 10 : 6 | Anſw. 162 yards, 5 feet. |

## OF RENDERING.

If the partitions between rooms be 141 feet 6 inches about, and 11 feet 3 inches high, how many yards do theſe partitions contain?

### EXAMPLE.

| F. I. | |
|---|---|
| 141 : 6 | 141.5 |
| 11 : 3 | 11.25 |
| 1556 : 6 | 7075 |
| 35 : 4 : 6 | 2830 |
| 9)1591 : 10 : 6 | 1415 |
| Anſw. 176 : 7 : 0 | 1415 |
| | 9)1591.875 |
| | 176.87 |

There are three things here which ſhould be particularly obſerved,

Firſt, If there are any doors, windows, or the like, in the partitioning, deductions muſt be made for them.

Secondly, When rendering upon brick-walls is meaſured, no deductions are to be made; but when rendering is meaſured between quarters, one fifth part may very well be deducted for the quarters, braces, and interſtices.

Thirdly, Whiting and colouring are both meaſured by the yard, as ceiling and rendering were; and likewiſe in whiting and colouring, one fourth, or at leaſt, one fifth part is to be added, as one fifth part is deducted, in rendering between quarters.

## CHAP. VIII.

### Of Plumbers Work.

*Diſcip.* WHAT method can be taken to meaſure Plumber's Work?

*Tyro.* No direct method can be taken; becauſe it is impoſſible, by the arts now in uſe, to caſt lead of an equal thickneſs throughout. And accordingly, all Plumber's Work is undertaken by the pound or hundred weight.

Many experiments have been tried to reduce this to ſome certainty, by meaſuring two parcels of the ſame dimenſions, and caſt at the ſame time; but they have all proved abortive; one of the parcels, though exactly of the ſame dimenſions, has always been found to be much heavier than the other. Beſides, ſome work requires a much greater quantity of ſolder than others, and conſequently muſt be dearer, the ſolder being of much more value than lead.

*Diſcip.* I am greatly obliged to you for this information; as I might otherwiſe have attempted, from meaſuring the ſurface, and the thickneſs in ſome particular part, to calculate the value of the lead covering any building; and by that means have expoſed myſelf to ridicule.

# BOOK II.

## *Of the* MENSURATION *of* SOLIDS.

*Discip.* WHAT am I to underſtand by the word Solid?

*Philo.* By a ſolid is meant a figure contained under three dimenſions, length, breadth, and depth or thickneſs; that is, the thickneſs of the body is here conſidered, which, in meaſuring ſuperficies is diſregarded.

To make this the more eaſy to be comprehended, let us ſuppoſe that a board is four inches thick; it is evident, that this board may be cut into four, the ſurface of each of which will meaſure the ſame as before the board was cut. But it would be very unreaſonable to ſuppoſe that a board of four inches thick ſhould be ſold at the ſame price per foot, as another which is only one. It is therefore often neceſſary to conſider the depth or thickneſs of bodies required to be meaſured; and this is what we mean by the menſuration of ſolids; or finding the content of bodies that have three dimenſions.

Solids are of various kinds; and therefore require various methods to find their contents.

1. If the ends and ſides are equal ſquares, the ſolid is called a cube.

2. If the baſes or ends are both equal, and both at right angles; the ſolid is called a parallelopiped.

3. It is called a cylinder when the ends are equal circles.

4. If the ſolid be formed on a triangular, ſquare, or polygonal baſe, having its ſides plain triangles, whoſe vertices all meet in the ſame point, it is called a pyramid; and is denominated from the number of ſides of which its baſis conſiſts; as triangular, ſquare, &c.

5. If the baſes, or ends, are equal, parallel, and ſimilar right lined plain figures, and the ſides parallelograms, it is called a priſm.

6. If the baſe be a circle, and the top or apex a point, it is called a cone.

7. If

7. If the ſolid be terminated by a convex ſurface, whereof every point is equally diſtant from a certain point within the ſolid, it is called a ſphere.

8. If the ſolid reſemble a ſphere, but one of its diameters be longer than the other; that is, if the ſuperficies be in the form of an oval, it is called a Spheroid.

Theſe are the principal figures, whoſe ſolid content is required to be found, and therefore I ſhall firſt, my dear Diſcipulus, explain the methods generally uſed for that purpoſe, before I proceed to lay down the rules neceſſary to be obſerved in meaſuring fruſtums, &c.

## *Of a* CUBE.

A Cube is a regular or ſolid body, conſiſting of ſix ſquare or equal faces or ſides, and its angles all right and therefore equal.

It is ſuppoſed to be generated by the motion of a ſquare plane along a line equal to one of its ſides, and at right angles thereto; whence it follows, that the planes of all ſections parallel to the baſe, are ſquares equal thereto, and conſequently equal to one another.

A cube being a ſquare ſolid, comprehended under ſix geometrical ſquares, in the form of a dye, the ſolid content of this figure is eaſily found by the following

## RULE.

Multiply the ſide of the cube into itſelf, and that product again by the ſide; the laſt product will be the ſolidity, or ſolid content of the cube.

## EXAMPLE.

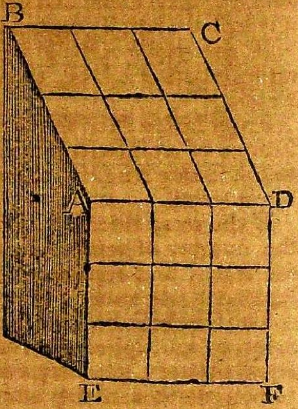
Suppoſe the ſide of a cube be ſeventeen inches and a half, what is the ſolid content?

OPERATION.

OPERATION.

```
     17.5
     17.5
    -----
      875
    1225
    175
   ------
   306.25
     17.5
  -------
   153125
  214375
  30625
 --------
 5359.375 the solid content.
```

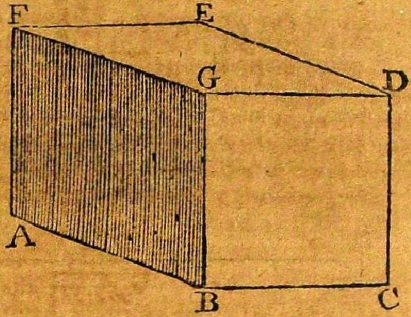
Suppose ABCDFE a cubical piece of stone, each side thereof being seventeen inches and a half, multiply 17.5 by 17.5, and the product is 306.25; which being multiplied by 17.5, the last product will be 5359 solid inches, and 375 parts.




To reduce the solid inches to feet, divide by 1728, because there are so many cubical inches in a foot, and the solid feet in the cube will be 3, and 175 cubical inches remain.

DEMON-

DEMONSTRATION.




If the ſquare FEDG be conceived to move down the plane AFGB always remaining parallel to itſelf, there will be generated by that motion a ſolid, having ſix planes; the two oppoſite of which will be equal and parallel to each other, whence it is called a parallelopipedon, or ſquare priſm.

If the plane ABGF be a ſquare equal to the generating plane FEDG, then will the generated ſolid be a cube.

From hence ſuch ſolids may be conceived to be conſtituted of an infinite ſeries of equal ſquares, each equal to the ſquare FEDG, and AF or BG will be the number of terms.

If the area, therefore, of FEDG be multiplied into the number of terms AF, the product is the ſum of all the ſeries, and conſequently the ſolidity of the parallelopipedon, or cube.

Or if the baſe FEDG, being divided into little ſquare areas, be multiplied into the height AF divided into an equal number, you may conceive as many little ſquares to be generated in the whole ſolid, as is the number of the little areas of the baſe, multiplied by the number of diviſions on the ſide AF.

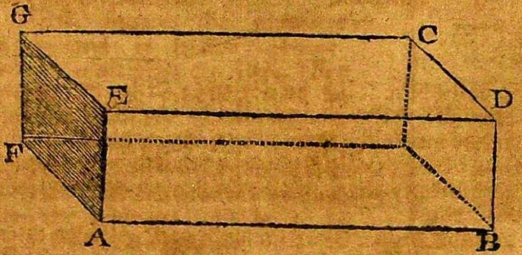
Thus if the ſide of the baſe FE be 10, that multiplied by itſelf is 100, which is the area of the ſquare baſe FEDG; then if AF be likewiſe 10, multiply 100 by 10, and the product will be 1000, and ſo many little cubes may this ſolid be cut into.

From this demonſtration it plainly appears, that if you multiply the area of the baſe of any parallelopiped into

into its height, that product will be the solid content of such a solid.

## *Of a* PARALLELOPIPED.

A parallelopiped is a solid figure, contained under six parallelograms, the opposite sides of which are equal and parallel; or it may be termed a prism, whose base is a parallelogram; and you will soon be shewn that a prism is always triple to a pyramid of the same base and height.




Let ABDCGFE be a parallelopipedon or a square prism, representing a square piece of timber or stone, each side of its square base AEGF, being 21 inches, and its length AB, 180 inches or 15 feet.

First then multiply 21 by 21, the product is 141, the area of the base in inches; which multiplied by 180, the length, the product is 79380, the solid content in inches.

Divide the last product by 1728, and the quotient is 45.9, that is, 45 solid feet, and 9 tenths of a foot.

Or thus, multiply 441 by 15 feet, and the product is 6615; divide this by 144 and the quotient is 45.9, the same as before.

Or thus, by multiplying feet and inches, multiply 1 foot 9 inches, by 1 foot 9 inches, and the product is 3 feet 0 inches, 9 parts; this multiplied again by 15 feet, gives 45 feet, 11 inches, 3 parts; that is 45 feet and $\frac{11}{12}$ of a foot, and $\frac{1}{4}$ of $\frac{1}{12}$.

See the Work.

| | | F. I. | |
|---|---|---|---|
| 21 | 441 | 1 9 | 1728)79380(45.9 |
| 21 | 15 | 1 9 | 6912 |
| 21 | 2205 | 1 9 | 10260 |
| 42 | 441 | 1 3 9 | 8640 |
| 441 | 144)6615(45.9 | 3 0 9 | 16200 |
| 180 | 855 | 15 | 15552 |
| 35280 | 1350 | 45 11 3 | 648 |
| 441 | 54 | | |
| 79380 | | | |

If the base of the ſolid be not an exact ſquare, or not in the form of a rectangled parallelogram, the method of meaſuring is much the ſame; for firſt you muſt find the area of the baſe by multiplying the breadth by the depth, and then multiply that area by the length of the piece as before: thus,

If a piece of timber be 25 inches broad, 9 inches deep, and 25 feet long, how many ſolid feet are contained therein?

| | | F. : I. |
|---|---|---|
| 25 | 144)5625(39 | 2 : 1 |
| 9 | 432 | 0 : 9 |
| 225 | 1305 | 1 : 6 9 |
| 25 | 1296 | 25 |
| 1125 | 09 | 25 : 0 0 |
| 450 | Anſw. 39 feet. | 12 : 6 0 |
| 5625 | | 1 : 0 6 |
| | | 0 : 6 3 |
| | | 39 : 0 9 |

## *Of a* Cylinder.

*Diſcip.* What ſpecies of ſolid is a cylinder?

*Philo.* A cylinder is a round ſolid, having its baſes circular, equal, and parallel, in form of a rolling ſtone, and the ſolid content is found by the following

Rule.

### RULE.

Multiply the area of the baſe by the length, and the product is the ſolid content.

### EXAMPLE.

Let ABC be a cylinder, whoſe diameter AB is 21.5 inches, and the length CD 16 feet, what is the ſolid content?

Firſt, ſquare the diameter 21.5, and it makes 462.25, which multiplied by .7854. (See page 28) the product will be 363.05115.

Then multiply this by 16, and the product will be 5808.8164. Divide this laſt product by 144, and the quotient will be 40.34 feet, the ſolid content.

*To find the ſuperficial Content.*

Firſt, find the circumference of the baſe 67.54; which being multiplied by 16, the product is 1080.64; which divided by 12, the quotient will be 90.05 feet, the curved ſurface; to which add 5.04 feet, the ſum of the baſes, and the ſum will be 95.09 feet, the whole ſuperficial content. See the work.

```
       67.54                                363.05
          16                                     2
   ---------                           -----------
       40524     90.05 }            144)726.10(5.04
       6754       5.04 } add              720
   ---------    -------                ---------
 12)1080.64      95.09                     610
    --------    -------                    576
       90.05                             -------
                                            34
```

## DEMONSTRATION.

The ſolid content of every cylinder is found by multiplying the area of its baſe into the height, as above; for every right cylinder is only a round priſm, being conſtituted of an infinite ſeries of equal circles; that of its baſe or end being one of the terms; and its height CD, in the figure, is the number of all the terms.

Therefore the area of its baſe AB being multiplied into CD, will be its ſolidity.

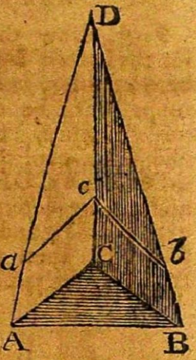
## Of a PYRAMID.

*Diſcip.* What do you mean by a Pyramid?

*Philo.* A Pyramid is a ſolid ſtanding on triangular, ſquare, or polygonal baſe; its ſides are plane triangles, and the ſeveral tops meet together in one point. Moſt ſteeples are pyramids: the properties of a pyramid are,

1. A triangular pyramid is the third part of a priſm ſtanding on the ſame baſe, and of the ſame altitude.

2. Hence as every multangular may be divided into a triangular body, every pyramid is the third part of a priſm ſtanding on the ſame baſe, and of the ſame altitude.




3. If a pyramid be cut by a plane a b c, parallel to its baſe ABC the former plane or baſe will be ſimilar to the latter.

4. All pyramids, priſms, cylinders, &c. are in a ratio compounded of their baſes and altitudes: the baſes therefore being equal, they are in proportion to their altitudes; and the altitudes being equal, they are in proportion to their baſes.

5. Pyramids, priſms, cylinders, cones, and other ſimilar bodies, are in a triplicate ratio of their homologous ſides.

6. Equal pyramids, &c. reciprocate their baſes and altitudes, that is, the altitude of the one is to that of the other, as the baſe of the one to that of the other. A ſphere is equal to a pyramid, whoſe baſe is equal in the ſurface, and its height to the radius of the ſphere.

7. The ſolidity of a pyramid is one third of the perpendicular altitude, multiplied by the baſe.

*To meaſure the ſurface and ſolidity of a Pyramid.*

Find the ſolidity of a priſm that has the ſame baſe with the given pyramid, and divide this by three; the quotient will be the ſolidity of the pyramid.

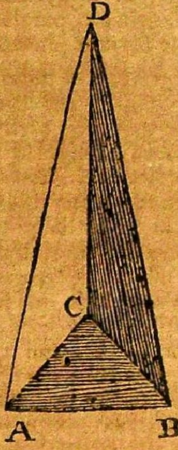
EXAMPLE.

Suppoſe the ſolidity of the priſm be found 6791031, the ſolidity of the pyramid will be thus found 2263677.

The external ſurface of a right pyramid ſtanding on a regular polygonian baſe, is equal to the altitude of one of the triangles which compoſe it, multiplied by the whole circumference of the baſe of the pyramid.

To

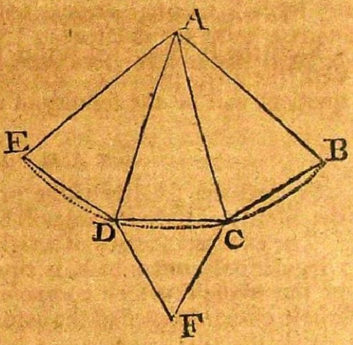
*To describe a Pyramid on a Plane.*




First, draw the triangle ABC for the base, if the pyramid required be triangular, so as that the side AB supposed to be turned behind, be not expressed. Secondly, on AC and CB, construct the triangles ADC, and CDB, meeting in any assumed or determined point, for example D; and draw AD, CD, and BD, then will ABCD be a triangular pyramid.

*To construct a Pyramid of Paste-Board, &c.*

Suppose it were required to construct a triangular pyramid.




1. With the radius AB, describe an arch BE, as in the figure, and thereto apply three equal chords BC, CD and DE.

2. On DC construct an equilateral triangle DFC, and draw the right lines AD and AC, this piece of paste-board, &c. being cut out from the centre of the figure, what remains within will turn up into a pyramid.

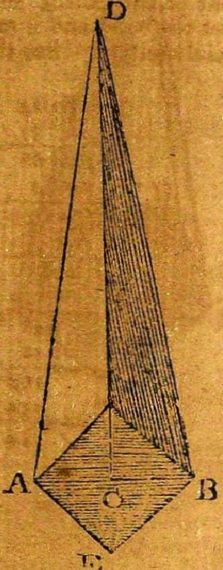
## *To find the solid Content of a Pyramid.*

### RULE.

Multiply the area of the base by a third part of the altitude, and the product will be the solid content of the pyramid.

### EXAMPLE.

Let ABCDE be a square pyramid, each side of the base being 18.5 inches, and the perpendicular height CD 15 feet: multiply 18.5 by 18.5, and the product is 342.25 the area of the base in inches, which multiply by 5, a third part of the height, and the product of that will be 1711.25, this being divided by 144, the quotient will be 11.88 feet.




```
        18.5
        18.5
      ------
         925
       1484
       185
      ------
      342.25 Area of base.
           5
      ------
144)1711.25(11.88 content
```

```
 F.   I.   P.
 1 :  6 :  6
 1 :  6 :  6
-------------
 1 :  6 :  6
      9 :  3
         9.3
-------------
 2 :  4 :  6.3
-------------
11 : 10 :  7.3
```

To

*To find the superficial Content.*

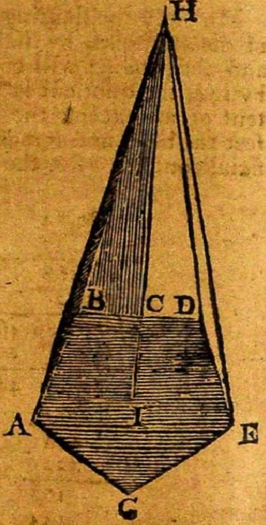
Multiply the slanting height, or perpendicular of one of the triangles, by half the periphery of the base 37, and the product will be 6668.88, which being divided by 144, the quotient is 46.31 feet, the superficial content of all except the base; then to that add 2.38 feet the base, and it makes 48.69 feet, the whole superficial content. See the work.

```
        180.24            144)342.25(2.38 nearly
             37                 288
       ------                  -----
        126168                   542
         54072                   432
       --------                 -----
144)6668.88(46.31               1105
        670    2.38             1152
       -----  ------
        908   48.69
        864
       -----
         448
         432
       -----
         168
         144
       -----
          24
```

EXAMPLE.

### EXAMPLE.

Let ABCDEGHI, be a pyramid, whoſe baſe is a heptagon, each ſide thereof being 15 inches, the perpendicular of the heptagon 15.58 inches, and the perpendicular height of the pyramid HI 13.5 feet; to find the ſolid and ſuperficial content.




### RULE.

Multiply 15.58, the perpendicular, by 52.5, half the ſum of the ſides of the heptagon, and the product will be 817.95, which multiplyed by 4.5, that is, $\frac{1}{3}$ of the height, and the product will be 3680.775.

Then divide this laſt product by 144, and the quotient will be 25.56 feet, the content.

15.58

```
        15.58 the heptagon's perpendicular.
         52.5 the half-sum of the sides.
       ------
         7790
        3116
        7790
       ------
       817.950
           4.5 a third part of the height.
       -------
       4089750
       3271800
     ---------
144)3680.7750(25.56 solid feet.
     288
     ---
     800
     720
     ---
      807
      720
      ---
       877
       864
       ---
        13
```

*To find the superficial Content.*

Multiply the height taken from the middle of one of the sides of the base 162.75 inches, by the half-sum of the sides 52.5 inches, and the product will be 8544.375; which divided by 144, the quotient will be 59.335 feet, the content of the upper part.

```
    162.75                              144)817.95(5.68
       52.5                                 720
    ------                                  ---
     81375                                   979
    32550                                    864
   81375                                     ----
   ---------                                 1154
144)8544.375(59.335 Feet                     1152
    720         5.68  Base add               ----
    ----     -------                            3
    1344     65.015 the whole content.
    1296
    ----
     483
     432
     ---
     517
     432
     ---
     855
     720
     ---
     135
```

## *Of a* PRISM.

*Discip*. What am I to underſtand by a Priſm?

*Philo*. A Priſm is a ſolid contained under ſeveral planes, and having its baſes alike, equal, and parallel. The ſolid content of a priſm, whether triangular or multangular, is found by multiplying the area of the baſe into the length or height, and the product will be the ſolid content.

EXAMPLE.

EXAMPLE.

Let ABCDEF be a triangular prism, each side of the base being 15.6 inches, the perpendicular of it CD 13.51 inches, and the length of the solid 19.5 feet. This and all other questions of the same kind may be easily solved by the following rule.

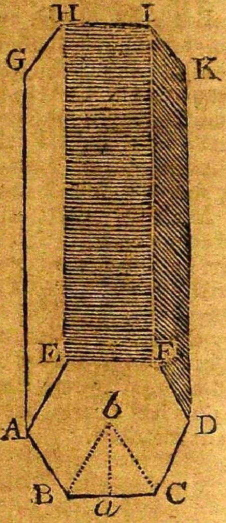
Multiply the perpendicular of the triangle 13.51, by half the base 7.8, and the product will be 105.378, the area of the base; which multiplied by the length 19.5, the product will be 2054.871; which divided by 144, the quotient will be 14.27 feet nearly, the solid content required.

```
   13.51
     7.8
  ------
   10808
   9457
  ------
 105.378
    19.5
  ------
  526890
 948402
105378
---------
2054.8710
```

```
144)2054.87(14.27
    144
    ---
     614
     576
     ---
     .388
      288
     ----
      1007
      1008
     -----
```

## EXAMPLE.

Let ABCDEFGHIK represent a prism, whose base is a hexagon, each side of which is 16 inches, the perpendicular from the center of the base to the middle of one of the sides, *ab*, 13.84 inches, and the length of the prism 15 feet; the solid content is required.




Multiply half the sum of the sides 48 by 13.84, and the product will be 664.32, the area of the hexagonal base; which multiply by 15 feet, the length, and the product will be 9964.80, which divide by 144, and the quotient will be 69.2 feet, the solid content required.

```
  13.84
     48
 ------
 11072
 5536
 ------
 664.32 Area of the base.
     15
 ------
 332160
 66432
 ------
 9964.80
```

```
144)9964.80(69.2
    864
    ----
    1324
    1296
    ----
     288
     288
     ---
     000
```

To find the superficial content of any of these solids, you must take the girt of the piece, and multiply by the length, and to that product add the two areas of the bases, and the sum will be the whole superficial content.

For

For example, in the hexagonal prism; the sum of the sides being 96, and the length 15 feet, that is 180 inches, multiply these into each other, and the product will be 17280 square inches, to which add twice 664.32 the areas of the two bases; and the sum is 18608.64, the area of the whole; which is equal to 129.22 feet.

```
  180              144)18608.64)129.22
   96                  144
 ----                  ---
 1080                   420
1620                    288
-----                   ---
17280                   1328
   664.32               1296
   664.32               ----
--------                  326
18608.64                  288
                          ---
                           384
                           288
                           ---
                            96
```

The superficial content of the whole solid is 129.22 feet.

## *Of a* Cone.

*Discip.* How do you define a Cone?

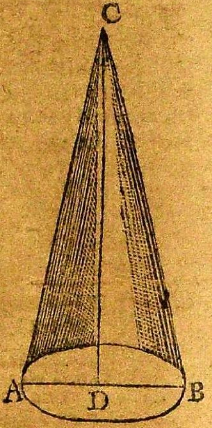
*Philo.* A Cone is a solid, having a circular base, and growing smaller and smaller, 'till it ends in a point, which is called the vertex, and may be nearly represented by a sugar-loaf. Its solidity will be found by the following

### Rule.

Multiply the area of the base by a third part of the perpendicular height, and the product is the solid content.

Let ABC be a cone, the diameter of whose base AB is 26.5 inches; and the height of the cone DC is 16.5 feet.

First square the diameter 26.5, and it will make 702.25, which multiplied by .7854, the product will be 551.54715; this being multiplied by 5.5 the product will be 3033.509325, which being divided by 144, the quotient will be nearly 21.066, the solid content of the cone.




```
144)3033.509, &c. (21.066 feet
    288                content
    ---
     153
     144
     ---
      950
      864
      ---
       869
       864
       ---
```

```
      26.5 the diameter
      26.5
      ----
      1325
     1590
     530
    ------
    702.25 Square
     .7854
    ------
    280900
   351125
  561800
 491575
 ---------
 551.54715 Area of the base
       5.5 1/3 Part of the height
 ---------
 275773575
275773575
-----------
3033.509325
```

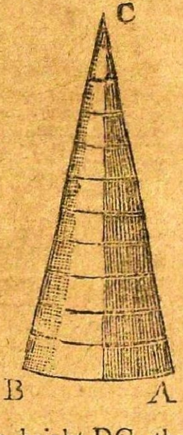
*To find the superficial Content.*

Multiply half the circumference 41.626 by the slant height AC 198.46, and the product will be 8261.09596; which

which being divided by 144, the quotient is nearly 57.37, the curve ſurface, to which add the baſe, and the ſum will be 61,2 feet, the ſuperficial content.

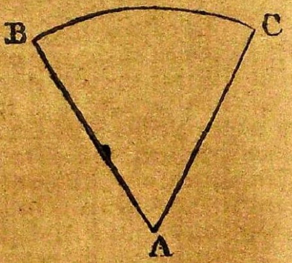
| | | |
|---|---|---|
| 41.626 | 144)8261.09596(57.37 | Feet nearly. |
| 198.46 | 1061 | 3.83 Baſe. |
| 249756 | 530 | 61,20 whole cont. |
| 166504 | 989 | |
| 333008 | 144)551.54(3.83 | |
| 374634 | 1195 | |
| 41626 | 434 | |
| 8261.09596 | 2 | |

## DEMONSTRATION.




Every cone is the third part of a cylinder of equal baſe and altitude. The truth of this may eaſily be conceived if we only conſider that a cone is but a round pyramid, and therefore it muſt needs have the ſame ratio to its circumſcribing cylinder, as the ſquare pyramid hath to its circumſcribing parallelopipedon, that is 1 to 3. However to make it clearer, let it be conſidered, that every right cone is conſtituted of an infinite ſeries of circles whoſe diameters continually increaſe in arithmetical progreſſion, beginning at the vertex or point C, the area of its baſe AB being the greateſt term, and its perpendicular height DC, the number of all terms, therefore the area of the circle of the baſe multiplied by a third part of the altitude DC, will be the ſum of all the ſeries, equal to the ſolidity of the cone.

The curve ſuperficies of every right cone is equal to half the rectangle of the circumference of its baſe into the length of its ſides.




For the curve ſurface of every right cone is equal to the ſector of a circle, whoſe arch BC is equal to the periphery of the baſe of the cone, and the radius AB equal to the ſlant ſide of the cone.

This will evidently appear, if you cut a piece of paper in the form of a ſector of a circle as ABC, and bend both the ſides AB and AC together till they meet, and you will find it to form a right cone.

## *Of a* SPHERE.

*Diſcip.* What ſpecies of ſolid do you call a ſphere; and how is it diſtinguiſhed from other ſolids?

*Philo.* A ſphere is a ſolid body contained under one ſingle ſurface, and having a point in the middle, called the center, whence all the lines drawn to the ſurface are equal.

The ſphere is ſuppoſed to be generated by the revolution of a ſemi-circle about its diameter, which is alſo called the *axis of the ſphere,* and the extreme points of the axis, the *poles of the ſphere.* And hence the following properties of the ſphere will be eaſily conceived.

### *Properties of the Sphere.*

1. A ſphere is equal to a cone, whoſe baſe is equal to the ſurface, and its height to the radius of the ſphere. Hence a ſphere being conſidered as ſuch a cone, its cube or ſolid content is found like that of a cone.

2. A ſphere is to a cylinder, ſtanding on an equal baſis, and of the ſame height as 2 to 3. Hence alſo may the cube or content be found.

3. The

3. The cube of the diameter of a ſphere, is to the ſolid content of the ſphere, nearly as 300 to 157 : and thus alſo may the content of the ſphere be meaſured.

4. The ſurface of a ſphere is quadruple to that of a ſphere deſcribed with the radius of a ſphere. For ſince a ſphere is equal to a cone, whoſe baſe is the ſurface, and its altitude the radius of the ſphere ; the ſurface of the ſphere is had, by dividing its ſolidity by a third part of its ſemi-diameter :

If therefore the diameter of the circle be 100, the area will be 7850 ; conſequently the ſolidity 1570000, which divided by a third of the ſemi-diameter 100, the quotient will be the ſurface of the ſphere, or 31400 ; which is manifeſtly quadruple the area of the circle, and hence the following

## RULE.

Find the periphery of the circle, deſcribed by the radius of the ſphere. This being found, multiply it into the diameter ; and the product is the ſurface of the ſphere. Multiply the ſurface by the ſixth part of the diameter, and the product will be the ſolidity of the ſphere.

## EXAMPLE.

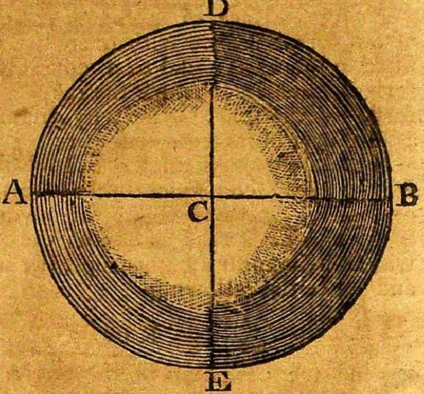
Suppoſing the diameter of the ſphere, 56, the periphery will be found 175 ; the product of theſe two multiplied by a ſixth part of the diameter, gives the ſolidity 92239. Or it may be done in the following manner.

Find the cube of the diameter 175616 : then ſay, as 300 is to 157, ſo is 175616 to a fourth proportional, *viz.* 92239, the ſolidity of the ſphere.

## Or thus ;

As 21 is to 11, ſo is the cube of the axis to the ſolid content. Or, as 1 is to .5236, ſo is the cube of the axis to the ſolid content.

EXAMPLE.




Let ABCDE be a globe, whose axis AB=DE is 20 inches: the circumference will be found to be 62.832: then by the first rule, multiply the circumference by the axis, and the product will be 1256.64, which is the superficial content in inches: take a sixth part of that sum, which is 209.44, multiply that sixth part by 20, the axis, and the product will be 4188.8, the solidity in inches.

Or if you multiply the superficial content by the axis, and take a sixth part of the product, the answer will be the same.

Or by this rule: The cube of the axis is 8000, which multiplied by 11, the product will be 88000, and divided by 21, the quotient will be 4190.47 the solidity nearly the same as before.

Or if the cube of the axis be multiplied by .5236, the product will be 4188.8, the solidity the same as by the first method, and nearly the same as above. If you divide 4188.8 by 1728, the quotient will be 2.424 feet.

62.832
20

6|1256.640 The superficial content.

209.44 A sixth part.
20

4188.80 Solidity in inches.

21 : 11 : 8000
11

21|88000|4190.47 The content.
40
190
100
160

13

1 : .5236 : : 8000
8000

1728|4188.8000|2.424 Feet, the solidity.

It should be observed, that if the axis of a globe be 1. the solidity will be .5236; and if the circumference be 1. the solidity will be .016887.

### *Of the* SPHEROID.

*Discip.* What do you call a Spheroid, and wherein does it differ from other solids?

*Phil.* A Spheroid is a solid approaching to the figure of a sphere, not exactly round, but oblong, as having one of its diameters longer than the other, and generated by the revolution of a semi-ellipsis about its axis.

When it is generated by the revolution of a semi-ellipsis about its greater or transverse axis, 'tis then called an oblong or prolate spheroid; and when generated by the revolution of an ellipsis about its lesser or conjugate axis, it is called an oblate spheroid.

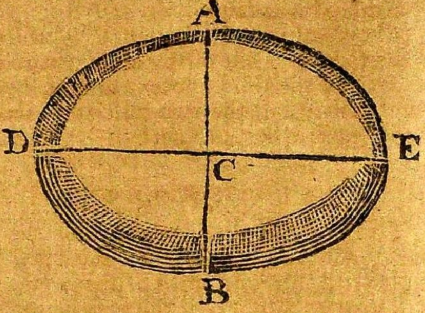
The solid dimensions of a spheroid is $\frac{2}{3}$ of its circumscribing cylinder; or it is equal to a cone, whose altitude is equal to the greater axis, and the diameter of the base, to four times the lesser axis of the generating ellipsis.

Or a ſpheroid is to a ſphere deſcribed on its greater axis, as the ſquare of the leſſer axis is to the ſquare of the greater. Or it is to a ſphere on the leſſer axis, as the greater axis is to the leſs. Hence the ſolid content of the ſpheroid may be found by the following RULE.

Multiply the ſquare of the leſſer diameter by the length, and that product multiply again by .5236; this laſt product will be the ſolidity of the ſpheroid.

## EXAMPLE.

Let AB, the conjugate diameter of the ſpheroid ABCDE, be 33 inches, and DE the length or tranſverſe diameter be 55 inches, and let the ſolidity be required.




## OPERATION.

```
  33
  33
 ----
  99
 99
 ----
 1089  Square of the
   55  conjugate diameter
 ----
 5445
5445
-----
59895

   59895
   .5236
  ------
  359370
 179685
119790
299475
----------
31361.0220 Solidity.
```

## DEMONSTRATION.

Every ſpheroid is equal to $\frac{2}{3}$ of a cylinder, whoſe baſe is equal to the circle deſcribed by the conjugate diameter

diameter of the ſpheroid, and its height equal to the length of the ſpheroid.

## *Of* MEASURING FRUSTUMS.

*Diſcip*. What do you mean by the word Fruſtum?

*Philo*. By the fruſtum of any figure is meant a piece cut off, and ſeparated from the body. Thus the fruſtum of a pyramid, or cone, is a part or piece of it cut off uſually by a plane parallel to the baſe.

I ſhall firſt ſhew you the method of meaſuring the fruſtum of a pyramid.

The fruſtum of a pyramid is the remaining part, when the top is cut off by a plane parallel to the baſe.

To find the ſolid content of this body, there are ſeveral rules; but the two following will be abundantly ſufficient for the purpoſe.

### RULE I.

To the rectangle or product, of the ſides of the two baſes, add the ſum of their ſquares; that ſum being multiplied into one third of the height of the fruſtum, will give its ſolidity, if the baſe be ſquare.

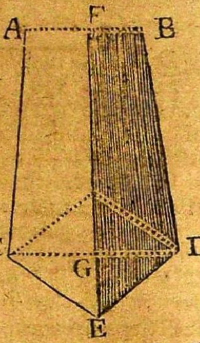
Or thus, which is the ſame in effect.

Multiply the areas of the two baſes together, and extract the ſquare root out of the product; that multiplied by one third of the height, will give the ſolidity of any fruſtum, either ſquare or multangled.

### RULE II.

Add one third part of the ſquare of their difference to the rectangle of the ſides of two baſes; and that ſum being multiplied into the height, will, if the baſes be ſquare, produce the ſolidity: But if they be triangular, or multangular, the ſaid rectangle of the ſides, with the third part of the ſquare of their difference, will be the ſquare of a mean ſide; and the ſquare root thereof will be ſuch a mean ſide, as will reduce the tapering ſolid to a priſm equal to it.

## As for Example.




Let ABCDEF be the frustum of a square pyramid, the side of the greater base 18 inches, and the side of the lesser base 12 inches, and the height FG 18 feet; what is the solidity of it?

First multiply the two sides together, 18 by 12, and the product will be 216; and the difference of the sides is 6, the square of which is 36; a third part of which is 12; which being added to 216, the sum is 228 inches, the area of a mean base; which being multiplied by 18 feet, the length, the product will be 4104. This being divided by 144, the quotient will be 28.5 feet, the content required.

### Or by the first Rule thus:

The square of 18 is 324, and the square of 12 is 144, and the rectangle of 18 by 12 is 216: The sum of these three is 684, which multiplied by 6, the product will be 4104; which divided by 144, the quotient will be 28.5 feet, the same as before.

The Operation both Ways.

```
   18     6 Diff.
   12     6
  ----------------
  216  3)36(Squ.
   12  add.
  ----------------
  228  the ſum.
   18  the height
  ----------------
  1824
  228
  ----------------
144)4104(28.5
    288
  ----------------
    1224
    1152
  ----------------
     ..720
     ..720
  ----------------
       ...
```

```
        18      12
        18      12
     ------------------
     { 324 Sq. 144 Square
     { 144
     { 216
     ------------------
       684 the ſum
         6 3d of the height
 144)4104(28.5 feet.
       288
     ------------------
       1224
       1152
     ------------------
        ..720
          720
     ------------------
           ...
```

By feet and inches thus :

```
            F. I. S.
Multiply    1 : 6 : 6
      by    1       6
            ---------
Product     1 : 6
Add         0 : 1
            ---------
Multiply    1 : 7
      by   18 : 0 height
            ---------
           18 : 0
            9 : 0
            1 : 6
            ---------
Content    28 : 6
```

Or thus ;

```
F. I.
2  3 Sq. of the greater.
1  6 the Rectangle.
1  0 Square of the leſs.
-----
4  9 Trip. of a mean area
6  0 a third of the height.
-----
28 6
```

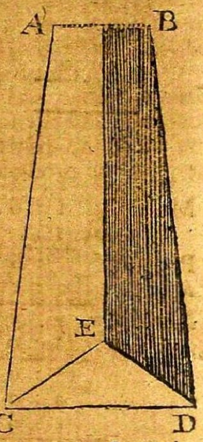
To find the ſuperficial Content.

The perimeter of the greater baſe is 72, and the perimeter of the leſſer baſe is 48 ; add both the perimeters together, the ſum will be 120 ; the half of this is 60 ; which being multiplied by 18 feet, the product will be 1080 ; this being divided by 12, the quotient is 90 feet,

feet, to which add the two bases 2.25 feet, and one foot, the sum will be 93.25 feet, the whole superficial content. See the following

### OPERATION.

| | |
|---|---|
| 18 12 | 18 Height |
| 4 4 | 60 |
| 72 48 | 12)1080( |
| 48 | 90 Feet |
| 2)120 | 2.25 the greater base |
| 60 | 1 the lesser base |
| | 93 25 Sum. |




Again, let ABCDE be the frustum of a triangular pyramid, each side of the greater base 30 inches, and each side of the lesser base 10 inches, and the length 20 feet: what is the solid content?

By the second rule multiply 30 by 10, and the product will be 300; and the difference between 30 and 10 is 20; which being squared, make 400, a third part of which is 133.333; which being added to 300, the sum is 433.333; and this being multiplied by .433, the product will be 186.633189, &c. the area of a mean base; and that multiplied by 20 feet, the length, the product will be 3732.66378. This divided by 144, the quotient will be 25.92127 feet, the solidity.

To find the superficial Content.

The perimeter of the greater base is 90, and the perimeter of the lesser base is 30; the sum of both is 120, and the half is 60, which being multiplied by 20 feet, the product will be 1200; which being divided by 12, the quotient will be 100; to which add the sum of the

the two bases 3.33 feet, and the sum will be 103.33 feet, the whole superficial content.

## *Of the* FRUSTUM *of a* CONE.

A frustum of a cone is the remainder of that solid, when the top end is cut off by a plane parallel to the base.

The solid content of it therefore is found in the same way as the frustum of a pyramid, regard being had to the nature of the lower and upper base, which here are circles.

### RULE I.

To the rectangle of the diameters of the two bases, add the squares of the said diameters, and multiply the sum by .7854, the product will be the triple of a mean area; which multiplied by one third of the perpendicular height, that product will be the solid content.

Or if you multiply the areas of the greater and lesser bases together, and out of the product extract the square root, and add the square root and two areas together, and multiply the sum by one third of the perpendicular height, the product will be the solid content.

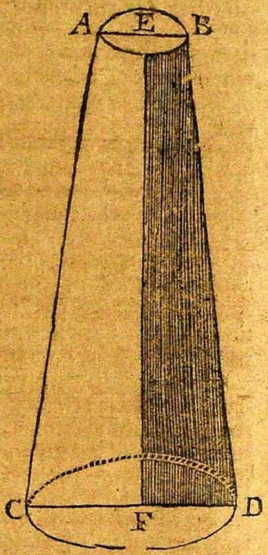
### RULE II.

To the rectangle of the greater and lesser diameters, add $\frac{1}{3}$ part of the square of their difference, and multiply the sum by .7854, the product will be a mean area, which multiplied by the perpendicular height, the product will be the solidity.

EXAMPLE.

## EXAMPLE.

Let ABCD be the frustum of a cone, whose greater diameter CD is 20 inches, and the lesser diameter AB 10 inches, and the height 20 feet, what is the solid content?




## OPERATION.

Multiply 20 by 10, and the product will be 200; and the difference between 20 and 10 is 10, whose square is 100, a third part of which is 33.33. which added to 200, the sum will be 233.33. This being multiplied by .7854, the product will be 183.257382, which being divided by 144, the quotient will be 1.273315 feet, the area of the mean base, which multiplied by 20 feet, the height, the product is 25.4663 feet, the solid content.

### To find the superficial Content.

You will find the circumference of the greater base to be 62.832, and that of the lesser base 31.416, the sum of both is 94.248, the half sum is 47.124; this multiplied by 20 feet, the product is 944.28; this divided by 12, the quotient is 78.54 feet, the curve-surface; to which add the sum of the areas of two bases 2.726 feet, the sum is 81.266 feet, the whole superficial content.

## *Of a* PRISMOID.

*Discip.* What do you mean by a Prismoid?

*Philo.*

*Philo.* By a prifmoid is meant the fruftum of a rectangled pyramid, when both bafes are parallel to one another but difproportional.

The content of this folid may be found by the following

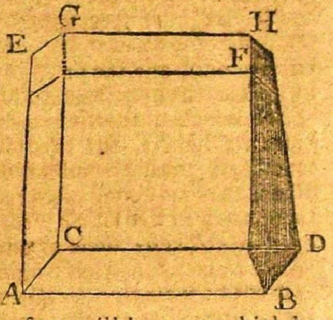
RULE.

Add to the greateft length, half the leffer length, and multiply that fum by the breadth of the greater bafe, and referve the product.

Then to the leffer length, add half the greater length, and multiply the fum by the breadth of the leffer bafe; add this product to the referved product, and multiply that fum by a third part of the height, the product will be the folid content.

EXAMPLE.

Let ABCDEFGH be a prifmoid given, the length of the greater bafe AB 40 inches, and its breadth AC 20 inches; the length of the leffer bafe GH 35, its breadth 15 inches, and the height 10 feet: What is the folid content?




To the greater length AB 40, add half the leffer length GH 17.5 the fum will be 57.5, which being multiplied by 20, the greater breadth, the product will be 1150, which referve.

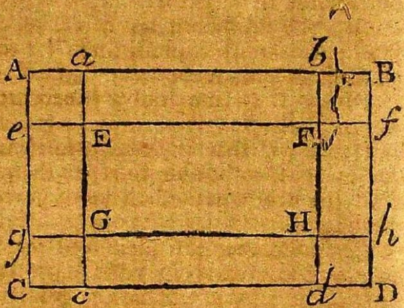
Then, to GH 35, add half AB 20, and the fum will be 55; which multiply by 15, the leffer breadth, and the product will be 825: To this product add 1150, the referved product, and the fum will be 1975; which being multiplied by 3.33, third part of the height, the product will be 6576.75; this product divided by 144, the quotient will be 45.67 feet, the folid content required.

To

## To prove this RULE.

Let the ſolid be ſuppoſed to be cut into pieces, ſo as to make it capable of being meaſured by the foregoing Rule

Let ABCD repreſent the greater and EF GH the leſſer baſe; and let the ſolid be ſuppoſed to be cut off by the lines *ac*, *bd*, *ef*, *gh*, from the top to the bottom; ſo will there be a parallelopipedon, having its baſes equal to the leſſer baſe EFGH, and its height 10 feet, equal to the height of the ſolid: multiply therefore 35 the length of the baſe by 15, the breadth of it, and the product will be 525; multiply this by 10 feet the height, and the product will be 5250.




Beſides this, there are two wedge-like pieces, whoſe baſes are *ab* EF GH *cd*; if theſe two pieces are laid together, the thick end of one to the thin end of the other, they will compoſe a rectangled parallelopipedon; there are alſo two other wedge-like pieces whoſe baſes *eg*, E*e*, F*f*, H*h*; theſe two laid together as before, will likewiſe compoſe a rectangled parallelopipedon. The content of theſe two parallelopipedons muſt be found in the ſame manner as that of other parallelopipedons; and their content, added to the above product, will ſhow the ſolid content of the whole.

## To find the ſuperficial Content.

Half the perimeter of the greater baſe is 60, and half the perimeter of the leſſer baſe is 50; which being added together, the ſum is 110; which being multiplied by 10 the height, the product will be 1000: this product divide by 12, and the quotient will be 83.33 feet;

feet; to which add the ſuperficial area of the two baſes 9.3 feet, and the ſum will be 92.63 feet, the whole ſuperficial content.

There are ſeveral other fruſtums belonging to different ſolids; but as they are of very little uſe, except in gauging, I ſhall not at preſent enumerate them; but reſerve them till I come to apply theſe rules to the finding the ſolid contents of various caſks, &c. when I ſhall take care to explain them in the eaſieſt and moſt conſpicuous manner.

*Diſcip.* I do not doubt your care and attention to my intereſt: I have already ſo greatly profited by your inſtructions, that my obligations to you are too great, for me ever to be able to repay them: and you are ſtill ſtudying every method in your power to increaſe their weight.

*Philo.* Perhaps the pleaſure I receive from ſeeing you improve ſo greatly in your learning, is at leaſt equal to yours, and it will be the greateſt ſatisfaction to ſee you, when I have finiſhed my inſtructions, readily meaſure any body whether ſuperfice or ſolid, with accuracy and diſpatch.

## *Of the regular* SOLIDS.

*Diſcip.* What do you mean by regular ſolids?

*Philo.* By regular ſolids is underſtood all thoſe bodies that are contained under regular and ſimilar planes, alike ſituated, and equally diſtant from the center.

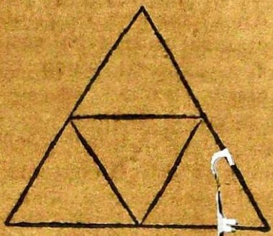
*Diſcip.* How many regular ſolids are there?

*Philo.* There are only five: the Tetraëdron; the Hexaëdron; the Octaëdron; the Dodecaëdron; and the Icoſaëdron.

*Diſcip.* What is a Tetraëdron?

*Philo.* A Tetraëdron is contained under four equal and equilateral plane triangles; forming four ſolid angles, each conſiſting of three triangles; and having ſix linear edges, and twelve plane angles.

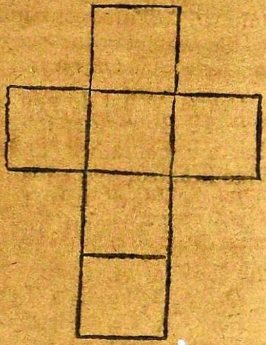
But



But you will more eaſily form an idea of the tetraëdron by drawing a figure, like that in the margin, on paſte-board, or any other pliable ſubſtance; for if you cut the lines half through, turn the parts up, and glue them together, they will form a complete tetraëdron; and as this is very eaſily done, I would recommend it to your attention.

*Diſcip.* You may depend upon my care to do every thing that is recommended by you; eſpecially when the performance has any tendency to facilitate my comprehending your inſtructions in a more complete manner.

*Philo.* I do not doubt your care; nor ſhould I have mentioned this particular, had it not tended to illuſtrate the ſubject in the plaineſt and moſt conſpicuous manner. But I muſt now return to the explanation of the reſt of the regular bodies.



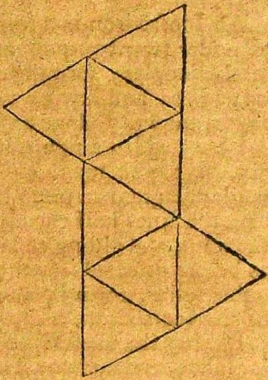
A Hexaëdron is a ſolid body contained under ſix equal ſquares, forming eight ſolid angles, each conſiſting of three ſquares; and having twelve linear edges and twenty four plane angles.

A ſimilar figure to that in the margin being drawn on paſte-board, cut half thro' in the lines, folded up, and glued together, will give you an adequate idea of an hexaëdron.

2

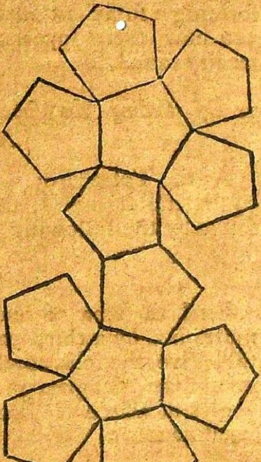
An Octaëdron is a ſolid contained under eight equal and equilateral plane triangles; forming ſix ſolid angles, each four triangles; having twelve linear edges, and twenty-four plane angles.

The figure in the margin drawn on paſte-board, the lines cut half thro' folded up, and glued together, will form the figure of an octaëdron.

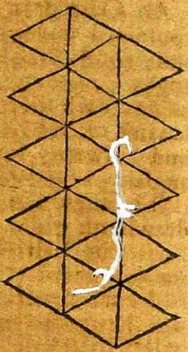


A Dodecaëdron is a ſolid figure contained under twelve equal, equilateral, and equiangular pentagons; forming twenty ſolid angles, each conſiſting of three pentagons, and having thirty linear edges, and ſixty plane angles.

If you draw a figure on paſte-board, ſimilar to that in the margin, cut the lines half through, fold the pentagons up, and glue the whole together; you will have the true figure of a Dodecaëdron.



An Icoſaëdron is a ſolid contained under twenty equal and equilateral triangles; forming twelve ſolid angles; and having thirty linear edges, and ſixty plane angles.



I have added a figure in the margin, that you may, by drawing a ſimilar diagram on paſte-board, cutting the lines half through, folding them up, and gluing them together, as before directed for the other regular ſolids, form an adequate idea of the Icoſaëdron.

Having thus deſcribed the regular ſolids, and laid down methods for forming a ſufficient idea of them, it remains to ſhew you how their ſuperficial and ſolid contents may be readily found.

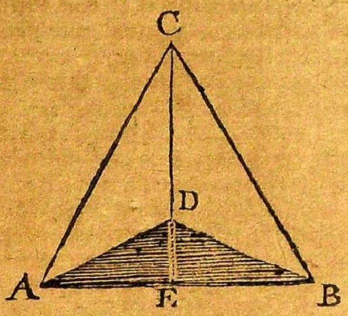
1. *Of finding the ſuperficial and ſolid Content of the* Tetraëdron.

As the Tetraëdron is contained under four equal and equilateral triangles, the ſolid content will be readily found by the following

RULE.

Find the area of one of the triangles. This area multiplied by a third part of the perpendicular height, will give the ſolid content required: And the area of one of the triangles, multiplied by the number of triangles of which the figure conſiſts, will be the ſuperficial content.

Let




Let ABCDE represent a Tetraëdron composed of four triangles, each equal to ABC: let AB=AC=CB, be 60 inches, and CD the perpendicular height of the figure be 51.96 inches and let the solid content of the Tetraëdron be required.

I have already pag. 17. shewn the method of finding the area of triangles, when their bases and perpendiculars are given; but as the triangles which form a Tetraëdron, are equilateral, or have their three sides equal to one another, it will be necessary to shew you, my dear Discipulus, how to find the perpendicular of an equilateral triangle, and this is easily done in the following manner. Square one of the sides of the triangle, as AB; and also the half of it, as AE. Subtract the lesser from the greater and extract the square root out of the remainder; and that root will be the perpendicular required.

```
Thus 60×60 —— 3600
And 30 × 30 —— 900
               ----
               2700(51.96 The perp.
               25
               ----
         101).200
              101
              ----
       1029).9900
             9261
             ----
     10386) ·63900
             62316
             -----
             ··584
             -----
```

The perpendicular being thus found, the area of the figure will be known by the rule given above.

OPERATION.

| | |
|---|---|
| 51.96. | The perpendicular |
| 30 | Half the base. |
| 1558.80 | Area of one triangle |
| 16.33 | One third of the height |
| 467640 | |
| 467640 | |
| 935280 | |
| 155880 | |
| 25455.2040 | The solid content. |

| | |
|---|---|
| 1558.8 | The area of one triangle |
| 4 | |
| 6235.2 | The superficial content. |

2. *Of finding the solid and superficial content of the* Hexaëdron.

The Hexaëdron is the same with what is otherwise called a cube, which I have already taught you how to measure in page 129, and therefore need not repeat it here.

3. *Of finding the solid and superficial content of the* Octaëdron.

The Octaëdron being contained under eight equal and equilateral triangles, or two quadrangular pyramids joined together by their bases, the solid content will be found by the following

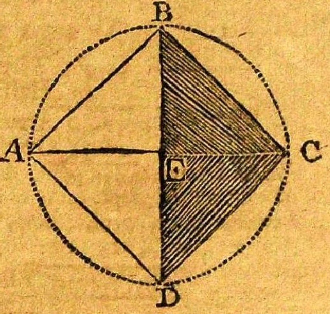
RULE.

Multiply the area of the base by a third part of the length of both pyramids, and the product will be the solid content required.

EXAMPLE

EXAMPLE.

Suppose ABCDE be an Octaëdron, whose side AB=BC CD=DA is 60 inches; what is the solid and superficial content?




OPERATION.

| | |
|---|---|
| 14.1421 | One third of the height. |
| 3600 | Area of the square base. |
| 84852600 | |
| 424263 | |
| 50911.5600 | The solid content required. |

The superficial content is easily found by the method laid down in the first problem.

OPERATION.

| | |
|---|---|
| 51.96 | Perpendicular of one triangle. |
| 30 | Half the base. |
| 1558.80 | |
| 8 | Numb. of triangles |
| 12470.40 | Superficial area. |

Or because the Octaëdron is composed of eight equilateral triangles, and the Tetraëdron of four; therefore the superficial area of the former is double to that of the latter.

OPERATION.

| | |
|---|---|
| 6235.2 | The superficial area of the Tetraëdron |
| 2 | |
| 12470.4 | Area of the Octaëd. the same as before. |

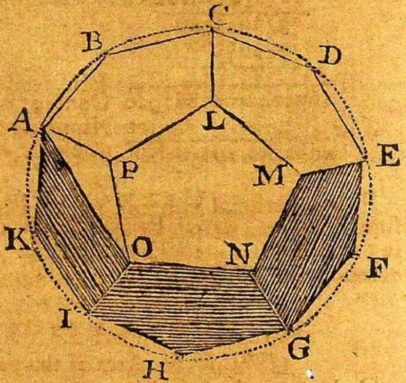
4. *Of finding the solid and superficial content of the* Dodecaëdron.

*Philo.* I have already, my dear Discipulus, informed you, that a Dodecaëdron is a solid body contained under twelve pentangular planes.

*Discip.* I well remember it; and have, pursuant to your instructions, drawn the figure on paste-board, you delineated for me, folded up the pieces, and by that means, procured an exact Dodecaëdron in miniature.

*Philo.* You have acted extremely right; and you will now have no difficulty in finding the superficial and solid content of this figure.

In order to this, let ABC DEFGHIKL MNOP represent a Dodecaëdron each side of which is equal to 60 inches; and let both the solid and superficial content be required.




The former or solid content of this figure is composed of twelve pentangled pyramids, whose vertices all meet in the center of the figure, and consequently, the first demand in the question may be easily answered by the following

RULE.

## RULE.

Find the ſolidity of one of thoſe pyramids; multiply that reſult by 12, and the product will be the ſolidity of the Dedecaëdron required.

The altitude of one of the pentangular pyramids, will be found to be 66.81095; and the perpendicular of the pentagon 41.29146.

```
      41.29146 Perpendicular
           360 Half the ſum of the ſides.
     ---------
      24778760
     12387438
     ---------
     14864.92560 Area of the pentagon.
        11.27032 A third part of 66.81095.
     -----------
      2972985120
     4459477680
  103254479200
   2972985120
  1486492560
 1486492560
 ---------------
 167524.468288192 0 Content of one pyramid.
                12 Number of pyramids.
 -----------------
 2012936.194583040 Solidity of the Dodecaëdron.
```

The ſuperficial content is eaſily had by finding the ſuperficial area of one of the pentagons, and multiplying that reſult by 12, the number of pentagons that compoſe the ſuperficies of the Dodecaëdron.

## OPERATION.

```
  14864.92560 Area of the pentagon.
           12
 ------------
 178379.10720 Superficial area required.
```

5. *Of finding the ſolid and ſuperficial content of the* Icoſaëdron.

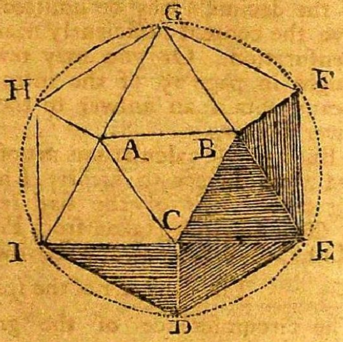
The Icoſaëdron, as I have already obſerved, is a ſolid body contained under twenty equal and equilateral triangles; and conſequently, its ſolidity is compoſed of twenty equilateral triangular pyramids, with their vertices all joining in the center. The ſolid content therefore is readily found by the following

RULE.

Find the content of one of theſe triangular pyramids; multiply that reſult by 20, and the product will be the ſolid content required.

EXAMPLE.

Let ABCDEFGHI be an Icoſaëdron, each ſide of which is 60 inches; and let both the ſolid and ſuperficial content be required.




OPERATION.

| | |
|---|---|
| 51.9612 | The perpendicular of the triangle |
| 30 | Half the ſide |
| 1558.8360 | Area of one triangle. |
| 20 | |
| 31176.7200 | Superficial content. |

Then

Then

```
         15.115228 One third alt. of the pyramid
          1558.286 Area of one triangle.
         ---------
          90691368
         45345684
       120921824
      120921824
      75576140
     75576140
    15115228
   ----------------
    23562.161554608
                 20
   ----------------
    47124.3231092160 The solidity.
```

As there is seldom any occasion for finding the solid content to so great a degree of accuracy, the major part of the decimals may be omitted; and the result will give the content sufficiently near the truth in actual mensuration. Or you may revert the multiplier as taught in page 25 of the introduction; and by that means obtain an answer to the question with much fewer figures.

But as these calculations are often very troublesome, and very subject to errors, I have added the following proportions of the five regular bodies inscribed in the same circle, taken from Peter Herigon. Cursus Math. vol. 1. p. 779, and Barrow's Euclid, lib. xiii.

The diameter of the sphere being 2.

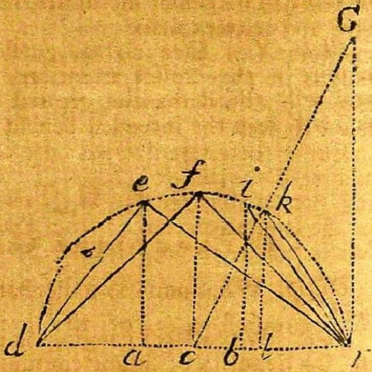
| | |
|---|---|
| The circumference of the greatest circle will be | 6.28318 |
| Superficies of the greatest circle ——— | 3.14159 |
| Superficies of the sphere ——— | 12.56637 |
| Solidity of the sphere ——— ——— | 4.1879 |
| Side of the Tetraëdron ——— | 1.62299 |
| Superficies of a Tetraëdron ——— | 4.61188 |
| Solidity of a Tetraëdron ——— | 0.15132 |
| Side of a cube or Hexaëdron ——— | 1.1547 |
| Superficies of the Hexaëdron ——— | 8. |

 Solidity

| | |
|---|---|
| Solidity of the Hexaëdron | 1.5361 |
| Side of an Octaëdron | 1.41429 |
| Superficies of the Octaëdron | 6.9282 |
| Solidity of the Octaëdron | 1.33333 |
| Side of the Dodecaëdron | 0.71364 |
| Superficies of the Dodecaëdron | 10.51462 |
| Solidity of the Dodecaëdron | 2.78516 |
| Side of the Icosaëdron | 1.05146 |
| Superficies of the Icosaëdron | 9.57454 |
| Solidity of the Icosaëdron | 2.53615 |

If one of these five regular bodies were required to be cut out of a sphere of any other diameter, it will be as the diameter of the sphere 2 is to the side of any one solid inscribed in the same (suppose the cube 1.1547) so is the diameter of any other sphere (suppose 8) to 9.2376, the side of the cube inscribed in this latter sphere.

Let *dr* be the diameter of any sphere, and *da* = *ab* = *br* one third of that diameter. Erect the perpendiculars *ae*, *cf*, and *bi*, and draw *de*, *df*, *er*, *fr*, and *ir*. Then will




1. *re*, be the side of the Tetraëdron.

2. *df* = *rf* the side of the Hexaëdron.

3. *de* the side of the Octaëdron.

4. Cut *de* in extreme and mean proportion in *h*, and *dh* will be the side of the Dodecaëdron.

5. Let the diameter *dr* be set up perpendicularly at *r*, and from the center *c*, to its top G, draw the line *c*G, cutting the circle in *k*, let fall the perpendicular *kl*; then is *lr* the side of the Icosaëdron.

BOOK

# BOOK IV.

## *The Application of the foregoing Rules to the Practical Mensuration of Solid Bodies.*

*Philo.* HAVING, my dear Discipulus, explained the theoretical part of the mensuration of solids; and traced out both by precepts and examples the path that must be pursued in all inquiries of this kind; it remains that I now shew you the best methods of appling these rules to practice.

*Discip.* The trouble you have already taken has laid me under the highest obligations: but I well know, that the only return I can possibly make, is to listen to your instructions, and, by that means, become a master of those branches of useful learning you have kindly undertaken to explain.

*Philo.* You have rightly guessed, Discipulus: all I require is the closest attention; for by that means, you will afford me the greatest satisfaction, because you will reap the intended benefit of my instructions.

## SECT. I.

### *Of* PRACTICAL GAUGING.

*Discip.* What am I to understand by practical gauging?

*Philo.* By practical gauging, is meant the art of finding the quantity of liquor contained in any cask, vessel, &c. And as these are of various forms, and consequently require various methods of computation, I shall begin with the easiest.

*To find the Area of any right-lined Superficies in Gallons,*

## PROBLEM I.

To find the area of any ſquare tun, back or cooler, &c. either in ale, wine, or corn gallons.

## RULE.

Multiply the given length, or breadth, they being in this caſe equal to each other, into itſelf, and the product will be the area in inches; then divide that area by 282, 231, or 268.8, and the quotient will be the area required.

6

## EXAMPLE.

Suppoſe the ſide of a ſquare tun, back or cooler, be 124.5 inches; what will its area be in gallons?

Firſt 124.5 multiplied by 124.5 gives 15500,25, the area in inches.

Then 282) 15500,25 (54,96, &c. the area in ale gallons.

And 231) 15500,25 (67,10, &c. the area in wine gallons.

Or 268.8) 15500,25 (57,66, &c. the area in corn gallons.

As the operations by diviſion are generally conſidered as more difficult than thoſe of multiplication, I will ſhew you, my dear, Diſcipulus, how you may readily change any diviſor into a multiplicator; and this requires nothing more than to divide unity or 1. by the diviſor you are deſirous of changing into a multiplicator, which will produce the ſame anſwer.

Thus 282) 1,000000(0,003546 the multiplicator for ale gallons.

231) 1,000000(0,004329 the multiplicator for wine gallons.

268.8) 1,000000(0,0037202 the multiplicator for corn gallons.

Hence

Hence you will easily find that 15500,25 multiplied by 0 03546, will give 54.96, the area in ale gallons as before, and so on for the rest.

## PROBLEM II.

To find the area of any tun, back or cooler, whose form is that of a right-angled parallelogram, in ale gallons, &c.

### RULE.

Multiply the length by the breadth, and the product will be the area in inches. This area being divided by the proper divisors, or multiplied by the equivalent multiplicators, will give the solid content in ale, wine, or corn gallons.

### EXAMPLE.

Suppose the length of a brewer's tun, back, or cooler be 217,5 inches, and its breadth 85,6 inches, what will its area be in ale or beer gallons?

First 217.5 multiplied by 85.6 gives 18648 the area in inches.

Then 282)18648(66,12, &c.

Or, $18648 \times 0,003546 = 66,12$, &c. the area required, &c.

## PROBLEM III.

To find the area of any triangular tun, back, or cooler, in ale gallons, &c.

### RULE.

Multiply the base by half the perpendicular, or half the base by the perpendicular, and the product will be the area required, and may easily be reduced to ale, wine, or corn gallons, in the same manner as before.

## EXAMPLE.

If the length of the baſe of a triangular cooler be 86,4 inches; and its perpendicular breadth be 57 inches; what will be its area in ale gallons?

Firſt, 86.4 the length multiplied by 28,5 half the perpendicular, gives 2462,4, the area in inches.

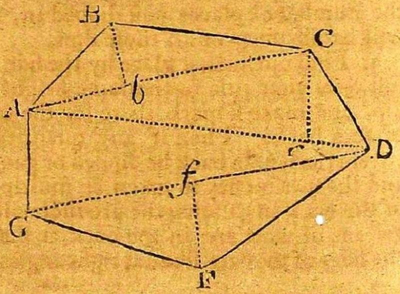
Then 282(2462,4(8,73, &c.

Or 24624×0,003546=8,73, &c. the area in ale gallons.

By proceeding in this manner, you will eaſily find the area of any tun, back or cooler, whether it be in the form a of Rhombus, Rhomboides, Trapezium, or of any other polygon, either regular or irregular, in ale, or beer gallons, &c. If you firſt divide it into triangles, and then find the areas of thoſe triangles: the ſum of thoſe areas being divided, or multiplied by its proper diviſor, or multiplicators, which will give the area required.

The practical method of dividing any polygonal tun, back, &c. into triangles, is by the help of a chalked line, like that uſed by carpenters, and may eaſily be performed in the following manner.

Suppoſe a brewer's tun, back or cooler to be in the form of the annexed figure ABCDFG. Let one end of the chalked line be faſtened with a nail, in any corner or angle of the back, as at A; then




ſtraining it to the angle at C, ſtrike the diagonal line AC, upon the bottom of the back; and ſtraining it again to the angle D, ſtrike another diagonal line, as AD: and

and ſo on for the diagonal line GD: Then having marked out all the diagonal, the perpendiculars may be thus found; faſten as before, one end of the chalked line in the angle B, and then by moving it to and fro upon the ſtretch, find out the neareſt diſtance between the angle at B, and the diagonal line AC, there ſtrike a line, and it will mark out the perpendicular B*b* from B to the line AC; proceed in the ſame manner to mark out the other perpendiculars, which being all drawn upon the bottom of the back, meaſure them, and each diagonal by a line of inches, &c. This done, the area of the back may be eaſily computed as directed by the method laid down in the laſt problem.

Having found the true area of the back or cooler, the next thing will be to diſcover the true dipping or gauging place in that back, that the true quantity of liquor may be computed or caſt up at any depth. This may be done in the following manner.

1. When the bottom of the back is covered all over, of any depth with liquor, then dip in your rule in eight or ten ſeveral places, more or leſs according to the largeneſs of the back, as remote and equally diſtant one from another as you well can, noting down the wet inches, and decimal parts of every dip.

2. Divide the ſum of all thoſe dips or wet inches by the number of places you dipped in, and the quotient will be the mean of all thoſe dips.

3. Find out ſuch a place by the ſide of the back, that juſt wets your rule to the ſame height with that mean dip, and make a mark there for the true dipping place of that back. Then if any quantity of wort which covers the whole back be dipped or gauged at that place, and the wet inches ſo taken be multiplied into the area of the back in gallons, the product will ſhew how many gallons of wort are in the back at that time, provided the ſides of the back ſtand at right angles with its bottom.

SEC-

## SECTION. II.

*To find the area of any circular and elliptical superficies in gallons.*

1. The periphery of the circle whose diameter is unity or 1. is 3,1416, and its area is 0,7854, see page 27 and 28.

2. The peripheries of all circles are in proportion one to another, as their diameters; and their areas are in proportion to the squares of their diameters.

As for Example; 1 : 3,1416 : : the diameters of any circle : to its periphery.

And 1 : 0,7854 : : the square of the diameter : to the area.

Upon these two proportions depends the solution of all the common or practical questions about a circle.

### PROBLEM.

The diameter of any circle being given, in inches, to find its area in gallons.

### RULE.

Multiply the square of the proposed diameter into 0,7854, and the product will be the area in inches. That area being divided by 282, or 231, &c. the quotient will be the area required.

### EXAMPLE.

Suppose the given diameter be 54·5 inches, what will be its area in gallons?

First, 54,5×54,5=2970,25 and 2970,25×0,7854=2332,83 the area in inches.

Then 282)2332,83(8,2724 the area in ale or beer gallons.

And 231)2332,83(10,0988 the area in wine gallons.

Or 268)2332,83(8,6788 the area in corn gallons.

But

But theſe areas in gallons may be much eaſier found, without calculating the circle's area in inches; by having the ſquare of the diameter of that circle whoſe area is one gallon; which may be eaſily found in the following manner.

0,785398 : 1 : : 282 : 359,05 the ſquare of the diameter of the circle whoſe area is 282 cubick inches, that is, one gallon:

And from this proportion will ariſe the following diviſors.

Viz. 0,785398)282,000000(359,05 will be a diviſor for ale gallons.

And 0,785398)231,000000(294,12 will be a diviſor for wine gallons.

Or 0.785398)268,800000(342,24 will be a diviſor for corn gallons.

If the ſquare of the diameter of any circle, be divided by any one of theſe conſtant or fixed diviſors, the quotient will ſhew that circle's area in their reſpective gallons. As for inſtance, ſuppoſe the ſquare of a circle's diameter is 2970,25.

Then 359,05)2970,25(8,2725 the area in ale gallons.

And 294,12)2970,25)10,0988 the area in wine gallons.

Or 342,24)2970,25(8,6788 the area in corn gallons.

But theſe diviſors may be eaſily turned into multiplicators, by dividing the area into inches of that circle whoſe diameter is 1.

That is, 0,785398 by 282; Or by 231, &c.

Thus 282)0,785398(0,02785 the multiplicator for ale gallons.

And 231)0.785398(0,003399 the multiplicator for wine gallons.

Or 268,8)0.785398(0,002922 the multiplicator for corn gallons.

Theſe multiplicators are the reſpective areas of a circle whoſe diameter is 1, and therefore if the ſquare of the diameter of any circle be multiplied with any of theſe numbers, the product will be that circle's area in gallons of the ſame name.

Viz. 2970,25×0,002785=8,2725 the area in ale gallons.

And 2970,25 × 0,003399 = 10,0938 the area in wine gallons, &c.

Thus, my dear Difcipulus, you will obferve, that if the diameter of any circle be given in inches, there are three feveral ways of finding its area in gallons, and all equally true; but that performed by the conftant divifors is moft commonly practifed.

## PROBLEM V.

The tranfverfe, or longeft diameter, and the conjugate, or fhorteft diameter of any elliptical fuperficies being given, to find its area in gallons.

### RULE.

Multiply the two diameters, viz. the length and breadth together, and divide their product by 359,05 for ale gallons, or 294,12 for wine gallons, &c. the quotient will be the area required.

### EXAMPLE.

Suppofe the longeft diameter to be 73,5 inches, and the fhorteft diameter to be 51,6 inches; what will the area be in ale gallons?

Firft, 73,5 × 51,6 = 3792,6. Then 359,05)3792,6(10,56 the area in ale gallons.

Or 294,12)3792,6(12,89 the area in wine gallons, &c.

## PROBLEM VI.

The diameter of any circle, and the verfed fine, viz. the height of any fegment being given; to find the area of that fegment in gallons.

In page 38, I have fhewn an eafy method of finding the area of any fegment of a circle; and muft now obferve, that if the area in inches found by that method, be divided by 282, or 231, &c. the quotient will be its area

in

in gallons; and because the area of any segment may be readily found in gallons, I have here added a short table, which will serve very well for common practice, not only to find the area of any segment of a circle in gallons; but also to find the number of gallons that are either drawn out, or remain in any cylindrick vessel along, or with its axis parallel to the horizon, usually called the ullage of a cask.

A TABLE

# A
# TABLE
## OF THE

Segments of a circle, whose area is unity or 1. The Diameter being divided by parallel chord lines into 100 equal parts.

| V.S. | Segm. | V.S. | Segm. | V.S. | Segm. | V.S. | Segm. |
|---|---|---|---|---|---|---|---|
| 1 | 0,0017 | 26 | 0,2066 | 51 | 0,5127 | 76 | 0,8155 |
| 2 | 0,0048 | 27 | 0,2178 | 52 | 0,5255 | 77 | 0,8262 |
| 3 | 0,0087 | 28 | 0,2292 | 53 | 0,5382 | 78 | 0,8369 |
| 4 | 0,0134 | 29 | 0,2407 | 54 | 0,5509 | 79 | 0,8474 |
| 5 | 0,0187 | 30 | 0,2522 | 55 | 0,5565 | 80 | 0,8576 |
| 6 | 0,0245 | 31 | 0,2640 | 56 | 0,5762 | 81 | 0,8677 |
| 7 | 0,0308 | 32 | 0,2759 | 57 | 0,5888 | 82 | 0,8776 |
| 8 | 0,0375 | 33 | 0,2878 | 58 | 0,6014 | 83 | 0,8873 |
| 9 | 0,0446 | 34 | 0,2998 | 59 | 0,6140 | 84 | 0,8968 |
| 10 | 0,0520 | 35 | 0,3119 | 60 | 0,6265 | 85 | 0,9058 |
| 11 | 0,0598 | 36 | 0,3241 | 61 | 0,6389 | 86 | 0,9149 |
| 12 | 0,0680 | 37 | 0,3361 | 62 | 0,6514 | 87 | 0,9236 |
| 13 | 0,0764 | 38 | 0,3406 | 63 | 0,6636 | 88 | 0,9320 |
| 14 | 0,0851 | 39 | 0,3611 | 64 | 0,6759 | 89 | 0,9402 |
| 15 | 0,0941 | 40 | 0,3755 | 65 | 0,6881 | 90 | 0,9480 |
| 16 | 0,1032 | 41 | 0,3860 | 66 | 0,7002 | 91 | 0,9554 |
| 17 | 0,1127 | 42 | 0,3989 | 67 | 0,7122 | 92 | 0,9692 |
| 18 | 0,1224 | 43 | 0,4112 | 68 | 0,7242 | 93 | 0,9692 |
| 19 | 0,1323 | 44 | 0,4238 | 69 | 0,7360 | 94 | 0,9755 |
| 20 | 0,1424 | 45 | 0,4265 | 70 | 0,7477 | 95 | 0,9813 |
| 21 | 0,1526 | 46 | 0,4491 | 71 | 0,7593 | 96 | 0,9866 |
| 22 | 0,1631 | 47 | 0,4618 | 72 | 0,7708 | 97 | 0,9913 |
| 23 | 0,1738 | 48 | 0,4745 | 73 | 0,7822 | 98 | 0,9952 |
| 24 | 0,1845 | 49 | 0,4873 | 74 | 0,7934 | 99 | 0,9983 |
| 25 | 0,1955 | 50 | 0,5000 | 75 | 0,8045 | 100 | 1,0000 |

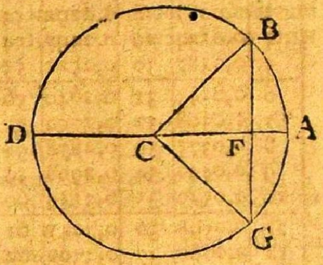
The

The uſe of this table of ſegments depends upon the following proportion.

As the diameter of any propoſed circle is to 100, (the diameter of the tabular circle): So is the height of any ſegment of a propoſed circle to a verſed ſine in the table.

Then if the tabular ſegment, which ſtands againſt that verſed ſine, be multiplied into the circle's area, either in inches or gallons, the product will be the area of the ſegment required, of the ſame name. If the circle's area be inches, the ſegment will be inches: If gallons, the ſegment will be gallons.

## EXAMPLE.




Let the diameter of the given circle be AD =62,5 inches; and the height of the ſegment ſought be FA = 20 inches; what will its area be in ale gallons?

Firſt, the area of the whole circle will be 10,8793 ale gallons, and the proportion will ſtand thus, 62,5 : 100 : : 20 : 32, the verſed ſine of the table, whoſe ſegment is 0,3759.

Then 10,8793×0,279=3,0016 ale gallons, being the area of the ſegment BAGF, as was required. The ſame method may be obſerved for wine gallons, corn gallons, or inches.

*Diſcip.* I ſhall find no difficulty in computing the area of any of theſe figures; but I ſhould be very glad to know how any ſuperficies can be ſaid to hold any number of gallons: It can contain no more of any fluid than what will cover its ſurface.

*Philo.*

*Philo.* Your obſervation is very pertinent and juſt; but it is always ſuppoſed in gauging that the fluid on any of theſe ſurfaces is one inch deep.

## SECT. III.

*To calculate the contents of ſuch veſſels, as are in the form of different ſolids, as priſms, parallelograms, &c.*

*Diſcip.* In what manner muſt I proceed to find the contents of veſſels of ſuch various forms?

*Philo.* Almoſt every different ſolid has its own particular rule, which muſt be followed in computation, as I have already explained in treating of the menſuration of ſolids, and ſhall therefore ſhew the method of applying thoſe rules to different veſſels by which their contents will be eaſily computed.

### PROBLEM VII.

To find the content of any priſm, whoſe ſides are parallelograms, what form ſoever its baſe is of.

That is, to compute the content, in gallons, of any tun, &c. whoſe ſides are parallelograms ſtanding upright, or at right angles with its bottom.

Firſt, find its ſolid content in inches by the rule laid down in page 140; then divide that content by 282 or 231, or 268,8, the quotient will ſhew the content in their reſpective gallons, viz. in ale, wine, or corn gallons.

Or multiply the conteht in inches by 0,003546, or 0,004329, &c. thoſe products will be the content in their reſpective gallons.

Or, you may purſue the following method; firſt find the true area of the tun's baſe or bottom; that area being multiplied by the tun's height, will produce the content in gallons, as before.

The work of this problem is ſo very eaſy as to require no example.

PRO-

## PROBLEM VIII.

To find the content of any pyramid, in gallons, whose base is bounded by right lines.

Every pyramid is one third part of its circumscribing prism?

Therefore if the area of any pyramid in gallons, be multiplied into one third of its perpendicular height, or if one third of that area be multiplied with the whole height, either of those products will be the content of the pyramid in gallons, &c.

The content of any square pyramid may be easily found in gallons by the following

### RULE.

Square the side of its base; and multiply that square by the perpendicular height; then divide that product by $846 = 282 \times 3$ for ale gallons; or by $693 = 231 \times 3$ for wine gallons; or by $806,4 = 268,8 \times 3$ for corn gallons; the quotient will be the content required.

Or if you multiply the above product by 0,001182 for ale gallons, or with 0,001443 for wine gallons, or lastly, with 0,001242, for corn gallons, the result will be the content required.

## PROBLEM IX.

*To find the content of the frustum of any square pyramid, cut off by a plane parallel to its base.*

First, either by rule one or two, given in page 153, find the proposed frustum's solidity in cubic inches. Then divide that content in cubic inches by 282. or 231, &c. and the quotient will be the content of the frustum in the respective gallons.

But from the methods already laid down, there may be easily deduced the following rule, for finding the content of the frustum of any pyramid, what form soever its bases are of, supposing them to be parallel, whether they are alike or unlike.

## RULE.

First find the area, then find a geometrical mean between those two areas, that is, multiply the two areas together, and extract the square root out of that product; the sum of those two areas, and their mean being multiplied into one third of the frustum's height, will produce the content required.

## EXAMPLE.

Suppose a tun in the form of the lower frustum of a pyramid, whose bases are equilateral triangles. Let the side of the top be 42 inches; the side of the bottom be 63,4 inches; and its depth be 33 inches; what will the content of that tun be in ale gallons?

First, find the area of each base in inches, then find what those areas are in ale gallons. Multiply those two areas together; the square root of their product will be the mean area, &c.

## EXAMPLE.

| | | |
|---|---|---|
| The area of the top is - | 2,71 | Ale gallons. |
| The area of the bottom is | 6,12 | |
| The mean area will be - | 4,07 | |
| | 12,90 | |

Then one third of 12,9×33=141,9 or one third of 12,9×33141,9 the content required.

## PROBLEM X.

*To find the content of any right cylinder in gallons.*

That is, to compute the content of any round tun, whose diameters at top and bottom are equal, and at right angles with its sides.

## RULE.

Multiply the square of the diameter into the height: and divide the product by 359,05, or multiply with 0,002785,

0,002785, &c. that quotient, or product, will be the content required.

### EXAMPLE.

Suppose the diameter be 42,5 and the height 31,3 inches. First 42,5×42,5=1896,25. And 1896,25× 31,5=56896,875. Then 35,05)56896,875(15846 the content in ale gallons, &c.

## PROBLEM XI.

*To find the content of any cone, or round pyramid in gallons.*

Every cone is one third of its circumscribing cylinder, see page 147. Therefore its content may be easily found by the following

### RULE.

Multiply the square of the diameter of its base into the perpendicular height; then divide their product by 1077,15=359,05×3 for ale gallons; or by 882,36= 294,12×3 for wine gallons, &c. and the quotient will be the content required.

Or if the product be multiplied by 0,000928= $\frac{0,002785}{3}$ or by 0,0011353=$\frac{0,0034}{3}$ those products will be the content in their respective gallons.

### EXAMPLE.

Suppose the diameter of the base be 42,5 and the perpendicular height be 31.5 inches; what will the content be in ale gallons?

First 42.5×42,5=1806,25. And 1806,25×31,5= 56896,875.

Then 1077,15)56896,875(52,82. or 56896,23×0000 928=52,82 the content in ale gallons; and in the same manner for wine or corn gallons.

PRO-

## PROBLEM XII.

*To find the content of the lower frustum of any cone, in gallons.*

That is, to compute the content of any round tun, whose diameter at top and bottom are parallel, but unequal.

The content of such a tun may be found by the rule page 187, whence it will be easy to deduce the following

### RULE.

To the triple product of the top and bottom diameters, add the square of their difference; multiply that sum into the height or depth, then divide the last product by 1077,15 for ale gallons, or by 882,36 for wine gallons, the quotient will be the content required.

### EXAMPLE.

Suppose the diameter at the top be 52.4 inches; the diameter at the bottom 44,6; and the height 30 inches, what is the content in gallons?

First 52,4×44,6=2337,04.
And 2337,04×3=7011,12
Also 52,4=44,6= 7.8
And 7,8×7,8= 60,84 } Add.

The height 30×7071,96=212158,8

Then 1077,15(212158,80(196,96
Or 112158,8×0.000928,=196,96 } Content in ale gallons.

You are to proceed in the same manner, either for wine or corn gallons as occasion requires. But if the tun or vessel be not truly circular; that is, if either its top or bottom, or both of them, be elliptical, whether they are alike or unlike is of no consequence, the content of such a tun may be truly found by the general rule, page 187.

## PROBLEM XIV.

The axis or diameter of any ſphere or globe being given in inches, to find its content in gallons.

Every ſphere is two thirds of its circumſcribing cylinder, by page 150 and from page 151 it appears, that if the cube of the axis of any ſphere, taken in inches, be multiplied into 0,5236, the product will be the content of the ſphere in cubic inches. Conſequently, if that content be divided by 282, or 231, &c. the quotient will be the content in gallons.

But thoſe two operations of multiplying with 0,5236 and the dividing by 282, or by 231, &c. may be contracted into one.

Thus 282(0,5236(0,001856 will be a multiplicator for ale gallons.

And 231)0,5236)0,002266 will be a multiplicator for wine gallons.

Or 0,5236(282(538,57 will be a diviſor for ale gallons.

And 0,5236)231(441,17 will be a diviſor for wine gallons.

### RULE.

If the cube of the axis of any ſphere be divided by 538,57, or multiplied with 0,001856, or divided by 441,17, or elſe multiplied by 0,002266; the quotient or product, will be the content of the ſphere in their reſpective gallons.

### EXAMPLE.

Suppoſe the axis or diameter of a ſphere or globe be 22 inches; what is its content in ale gallons?

### OPERATION.

Then $22 \times 22 \times 22 = 10648$ and 538,57) 10648 (19,76 ale gallons.

Or $10648 \times 0,001856 = 19,76$ ale gallons, the content required.

And ſo for either wine or corn gallons, as occaſion requires.

## PROBLEM XV.

To find the content of any ſegment of a ſphere, in gallons.

If the diameter of the ſegment's baſe, and its height are given; the content may be found by the following rules:

### RULE I.

To triple the ſquare of half the diameter, add the ſquare of the height; then multiply that ſum into the height, and divide the product by 538,57 for ale gallons; or by 441,17 for wine gallons, &c.

But if the axis of the ſphere, and the height of the ſegment are given; the content may be found by the next rule.

### RULE II.

From triple the product of the axis into the height ſubtract twice the ſquare of the height; then multiply the remainder into the height, and divide that product by 538,57, &c.

Either of theſe rules will produce the content of the ſegment in gallons.

### EXAMPLE.

Suppoſe the diameter of the ſegment's baſe be 28 inches, and its height be 8 inches; what will it contain in ale gallons?

### OPERATION.

Firſt 2) 28 (14. Then (per Rule 1) $14\times14\times3=588$, and $6\times6=36$. Next $588+36=624$. Again, $924\times6=3744$. Laſtly, 538,57)3744(695 the content required.

## SECT. IV.

*The practical Method of gauging any fixed tun or copper, and making a table to ſhew what it will hold at every inch deep; uſually called inching of a tun,* &c.

The tuns of brewers in general are ſo fixed, as to lean a little for conveniency of cleanſing their drink, which is uſually called the drip, or fall of the tun. The drip or fall of any tun, is the hoof of ſuch a ſolid as that tun is ſuppoſed to repreſent, and may be found by an algebraical proceſs: but the practical method is, to meaſure into the tun, ſo much liquor as will juſt cover its bottom; by which means you find at once both the fall and a true horizontal line over the bottom of the tun, from which, if the depth of the tun, (viz. the neareſt diſtance from the top of the tun to the ſurface of the liquor) be ſet off upon every one of its ſides, you will then have a true parallel plane at the top of the tun to that of the liquor.

Then if the ſides of the tun are ſtraight from the top to the bottom, take as many dimenſions in the two planes, as are neceſſary to find the true area of each; and by thoſe two areas, and the above depth, find ſo much of the tun's content, as is contained between thoſe two planes.

Next to inch that tun; divide the difference between the top and bottom areas by the depth, and the quotient will be an addend or fixed number, which being added to the leſſer area, the ſum will be the area of the next inch, and being added to that area, their ſum will be the area of the third inch, and ſo on from inch to inch, until the area of every ſingle inch be found; the ſum of thoſe areas, if the work be true, will be equal to the content found as above. And if the tun's drip or fall be added to the ſum of all thoſe areas, that ſum will be the whole or full content of the veſſel.

From what has been obſerved it is ſufficiently evident, that if 1, 2, 3, or any number of thoſe areas

counted from the bottom be added to the fall, that ſum will ſhew the quantity of liquor that is in the tun, to ſuch a number of wet inches from the bottom, as there are areas added together.

Or if the ſum of any number of thoſe areas, being counted from the top, be ſubſtracted from the tun's whole content, the remainder will ſhew what quantity of liquor is in the tun, when there is ſuch a number of dry inches from the top as there are areas ſubſtracted.

When theſe directions are well conſidered there will be no difficulty in forming a table to every wet or dry inch of any tun, whoſe ſides are ſtraight from the top to the bottom; what form ſoever its baſes are of, and whether it ſtand upon its greater or leſſer baſe.

But if the ſides of the tun are not ſtraight from the top to the bottom; then the beſt and eaſieſt way will be to divide the tun into ſeveral fruſtums, each of 10 inches deep, and finding the content of every ſingle fruſtum, by taking the diameters in the middle of every one of thoſe 10 inches; that is, the firſt diameter at five inches from the top; the ſecond diameter at 15 inches from the top, &c. and multiplying their reſpective areas by 10, which is done by only removing the ſeparating commas one place forward to the right hand. This being done, if the ſum of all thoſe fruſtums be added to the fall, that ſum will be the whole content of the tun.

It muſt be obſerved, that if you take the height of the 10 inch fruſtums in the ſide of the tun, you muſt allow for the difference between the ſlant height, and the perpendicular height in every fruſtum.

If from the whole content of the tun, you ſubtract the mean area of the firſt fruſtum ten times, and from the remainder ſubtract the mean area of the ſecond fruſtum ten times, and from the laſt remainder ſubtract the mean area of the third fruſtum, &c. until there remains nothing but the fall or hoof of the tun; you will have a table that will ſhew what quantity of drink is in the tun, to any number of dry inches.

This method is likewiſe uſed for gauging and inching a brewer's copper. Firſt, Meaſure into the copper

as much liquor as will juſt cover its crown: then divide its perpendicular height into fruſtums, and its ſides into four equal parts, that ſo croſs diameters may be taken in the middle of each fruſtum. But if the copper is much wider at the top than at the bottom, and its ſides ſpheroidical arching; then inſtead of taking thoſe mean diameters in the middle of every ten inches, you muſt take them in the middle of every ſix inches; but in every other reſpect you may proceed as before.

But the quantity of liquor that would cover the crown of the copper may be found without meaſuring it as above, by ſuppoſing the crown to be the ſegment of a ſphere, and the lower part of the copper, wherein the crown riſes, to be the fruſtum of a parabolic conoid: then if the diameter at the top of the crown, and its perpendicular height are given, the quantity of liquor may be found by the following

### Rule.

From the area of the plane at the top of the crown ſubſtract $1\frac{1}{3}$ of the area of the crown's height; the remainder being multiplied into half the height of the crown, will produce the quantity or number of gallons that will cover the crown.

## SECT. V.

*To compute the content of any cloſe caſk, viz. butt, pipe, hogſhead, &c. in gallons.*

In order to perform this difficult part of gauging, the three following dimenſions of the propoſed caſk muſt be accurately taken in inches, and decimal parts of an inch.

Viz. The bouldge or bung diameter, within the caſk.

Either of the head diameters, ſuppoſing them both equal.

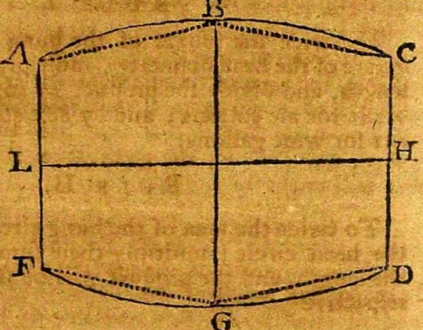
And the length of the caſk within.

In taking theſe dimenſions, it muſt be carefully obſerved, firſt, that the bung-hole be in the middle of the caſk; alſo that the bung-ſtaff, and the ſtaff over againſt the

bung-hole, are both regular or even within. Secondly, That the heads of the caſk are equal and truly circular; if ſo, the diſtance between the inſide of the chine, to the outſide of it's oppoſite ſtaff, will be the head diameter within the caſk. Thirdly, With a ſliding pair of calipers, take the ſhorteſt diſtance between the outſides of the two heads; from that length ſubſtract $1\frac{1}{2}$ inch more, or according to the largeneſs of the caſk, for the thickneſs of the head; the remainder will be the length of the caſk within.

Now, by theſe dimenſions it is natural to ſuppoſe, the content of the caſk were perfectly limited; but it will be eaſy to perceive, by the following figure, that the diameters and the length of one caſk, may be equal to thoſe of another; and yet one of thoſe caſks may contain ſeveral gallons more than the other.

For inſtance, ſuppoſe the annexed figure ABCDGF, to repreſent a caſk; then it is plain, that if the outward curved lines ABC, and FGD are the bounds or ſtaves of the caſk; it muſt of courſe hold more than if the inner ſtraight or pricked were its bounds or ſtaves; and yet the bung diameter BG, head diameters CD and AF, and the length LH, are the ſame in both thoſe caſks.




From this it plainly appears, that no one certain or general rule can be preſcribed, to find the true content of all ſorts of caſks; and therefore, gaugers uſually ſuppoſe every caſk to be in form of ſome one of theſe following ſolids.

*Viz.*
- I. The middle zone or frustum of a spheroid.
- II. The middle zone or frustum of a parabolic spindle.
- III. The lower frustum of two equal parabolic conoids.
- IV. The lower frustum of two equal cones.

The method of finding the form of the cask, and computing its content according to that supposed form, is to be performed thus:

I. If the staves of the cask are very curved or arching, as the outward lines of the last figure; then the cask is supposed to be in the form of the middle zone, or frustum of a spheroid, whose content may be computed by the two following rules.

### RULE I.

To twice the square of the bung diameter, add the square of the head diameter; multiply that sum into the length, and divide the product by 1077,15. viz. 3,8197 ×282 for ale gallons; and by 882,36. Viz. 3,8197× 231 for wine gallons.

### RULE II.

To twice the area of the bung circle, add the area of the head circle; multiply their sum into one third of the length, and the product will be the content in their respective gallons.

### EXAMPLE I.

Suppose a cask in the form of the middle zone of a spheroid, whose bung diameter is 31,5, head diameter 24,5, and its length 42 inches.

First, 31,5×31,5×2= 1984,5 and 24,5×24,5= 600,25.

Again, 1984,5×600,25=2584,75. and 2584,75×42 =108559,5.

Then 1077,15)108559,5(100,78 the content in ale gallons.

 And

And 882,35)108559,5(123,07 the content in wine gallons.

Or thus by the second Rule.

| | |
|---|---|
| Bung diameter 31,5, twice its circle's area is | 5,5270 |
| Head diameter 24,5, its circle's area is | 1,6718 |
| The length 42 divided by 3 is 14. Their sum | 7,1988 |

Then 7,1988×14=100,78 the content in ale gallons.

The content in wine gallons may be found in the same manner.

II. If the staves of the cask are not quite so much curved or arching as was supposed before; the cask is then taken for the middle frustum of a parabolic spindle; and its contents may be computed by this

## RULE.

To twice the square of the bung diameter, add the square of the head diameter; from their difference substract four tenths of the square of the difference of the diameters; multiply the remainder into the length, and divide the product by 1077,15, &c. as above.

## EXAMPLE II.

Suppose the dimensions the same as before. Then 31,5×31,5×2: +24,5×24,5=2584,75, and 31,5—24,5 =7. Again, 7×7=49 four tenths of which =19,6. and 2584,75—19,6×42=107736.3. Then 1077,15) 107736,3(100,10 the content in ale gallons, &c.

III. When the staves of the cask are but very little curved or arching, then it is supposed to be in the form of the frustums of two equal parabolic conoids, abutting or joining together upon one common base at the bouldge, and the content may be found by the following rules.

## RULE I.

To the square of the bung diameter, add the square of the head diameter; multiply their sum into the length, and divide the product by 718,08.

Viz.

Viz. 2,5464×282) for ale gallons; or by 588,22. Viz. 2,5464×231) for wine gallons; or thus:

### Rule II.

To the area of the bung circle, add the area of the head circle, multiply the ſum into half the length, and the product will be the content required.

### Example III.

With the ſame dimenſions as before.

Firſt, 31,5×31,5:+24,5×24,5=1591,5 and 1592,5 ×42=66885. and 718,08)66885(93,01 the content in ale gallons.

Or 588,22)66885(113,7 the content in wine gallons.

IV. If the ſtaves of the caſk are ſtraight from the bouldge to the head, as in the inner pricked lines in the laſt figure; it is then taken for the lower fruſtums of two equal cones, abutting or joining together upon one common baſe at the bouldge, and its contents may be computed thus:

### Rule.

To the ſum of the ſquares of the head and bung diameters, add their product; then multiply that ſum into the length, and divide the laſt product by 1077,15. Or by 882,36, the quotient will be the content, &c.

### Example IV.

With the ſame dimenſions as before.

Firſt, 31,5×31,5:+24,5×24,5:+31,5×24,5=2364, 25. And 2364,25×42=99298,5. Then 1077,15) 99298,5(92,18, the content in ale gallons, and ſo on for wine gallons.

Thus you have the methods of computing the true contents of the four ſolids, in whoſe forms all caſks

are ſuppoſed to be; and by the examples it appears, that of four ſuch caſks as have their dimenſions all equal, and the ſame with thoſe above-mentioned, the contents will be as in the margin.

| | Ale Gall. | Differ. |
|---|---|---|
| I. | 100,78 | 0,77 |
| II. | 100,01 | 7,00 |
| III. | 93,01 | 0,83 |
| IV. | 92,18 | |

From the diſproportion or inequality of theſe differences, it will be eaſy to imagine, that there may be ſeveral caſks whoſe contents cannot be truly found, according to the ſuppoſed forms; and, therefore, in order to rectify thoſe inequalities, ſome authors have laid down theorems of their own invention, and others have propoſed tables for the ſame purpoſe; but ſince all that we can attain to is only gueſs at the truth, the beſt and eaſieſt way is to be preferred in practice, which is, by finding ſuch a mean diameter as will reduce the propoſed caſk to a cylinder, thus:

Multiply the difference between the head and bung diameters by 0,7 or by 0,65 or by 0,6, or by 0,55 according as the ſtaves of the caſk are more or leſs arching; add the product to the head diameter, and the ſum will be the mean diameter required.

## EXAMPLE V.

With the ſame dimenſions as before. Then the bung diameter leſs the head diameter is 31,5—24,5=7. And

| | M. D. | A. G. | Cont. | Dif. |
|---|---|---|---|---|
| 24,5 | 7×0,7=29,4 its area | 2,4073×42= | 101, 0 | 2,39 |
| | 7×0,65=29,05 ——— | 2,3504×42= | 98,71 | 2,36 |
| | 7×0,6=28,7 ——— | 2,2941×42= | 96,35 | 2,32 |
| | 7×0,55=28,35 ——— | 2,2385×42= | 94,03 | 2,32 |

Hence it may be obſerved, that the differences between each caſk's content is regular, and very near equal; which plainly ſhews, that there is not ſo much room left in error by this method of computing their contents, as by the above forms.

SECT.

## SECT. VI.

To find what quantity of liquor is either drawn forth, or remaining in any ſpheroidical caſk, uſually called the ullage of a caſk.

CASE I. To find what quantity of liquor is in the caſk, when its axis is perpendicular to the horizon; viz. when it ſtands upright upon one of its heads.

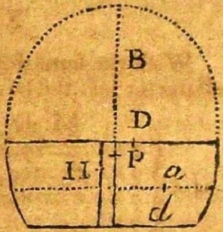
In order to perform this the eaſieſt way, it will be convenient to know how to calculate the area of any circle, betwixt the bung and the head, whoſe diſtance from the bung or middle of the caſk is given, which may be done in the following manner.

As the ſquare of half the length of the caſk : is to the difference between the bung and head areas : : ſo is the ſquare of any circle's diſtance from the bung : to the difference between the bung area, and the area of that circle; viz. the area of the liquor's ſurface.

### DEMONSTRATION.

Let { H = half the length of the caſk. D = half the bung diameter. *d* = half the head diameter. }

And { P = the diſtance of any circle from the bung. *a* = half the diameter of that circle. }




Then, according to the common property of the ellipſis, it will be, $BB : : DD . : : BB - HH : dd$. And $BB : DD : : BB - PP : aa$.

Hence this analogy $HH : DD - dd : : PP : DD - aa$. Then $DD - aa$ being ſubtracted from $DD$, will leave $aa$: but circles areas are in proportion to the ſquares of their diameters. Therefore, the above analogy is true.

 Then,

Then, from the bung area, ſubtract one third part of the aforeſaid difference; viz. between the bung area, and the area of the liquor's ſurface; multiply the remainder by the liquor's diſtance from the bung, and the product will ſhew what quantity of liquor is either above, or under half the content of the caſk.

## EXAMPLE.

Let us ſuppoſe a caſk of the ſame dimenſions with that in the firſt example, page 126, and let it be required to find what quantity of liquor, ale meaſure, is in it, when there is but nine inches wet. Here half the length of the caſk is 21 inches, whoſe ſquare is 441, and the liquor's diſtance from the bung is 21—9=12, its ſquare is 144. The difference between the bung and head areas is 1,0917(=2,7635—1,6718). Then 441 : 1,0917 : : 144 : 0,3564.

And 2,7635—0,3564=2,4071, the area of the liquor's ſurface.

Again, 3)0.3564(0,1188. And 2,7635—0,1188= 2,6447. Then 2,6447×12=31,7364, what the caſk wants of being half full. Conſequently 50,39--31,73= 18,66 will be the quantity of liquor in the caſk at nine inches wet, in ale gallons.

And if the caſk had wanted but nine inches of being full; then 50,39×31,73=82,12 would have been the quantity of liquor in the caſk.

As the two firſt terms, 441 and 1,0917, in the proportion, are fixed, or continue the ſame for any diſtance; it will be eaſy to calculate the areas of all the circles between the bung and head to every inch; and by that means to make a table which will ſhew what quantity of liquor is either drawn out or remaining in the caſk at any depth.

CASE 2. To find what quantity of liquor is in any caſk, when its axis is parallel to the horizon; viz. when it lies along.

The trueſt method of coming to the truth, in all ſorts of caſks, is by computing the ullage by a table of ſegments, in the following manner:

1, By

1. By the bung and head diameters find ſuch a mean diameter as you judge will reduce the propoſed caſk to a cylinder, by the method already laid down; and then find its full content by the preceding examples.

2. From the bung diameter, ſubtract the mean diameter, and halve their difference, that is, divide it by two.

3. From the wet inches of the propoſed ullage, ſubtract the ſaid half difference, and call it $x$; then obſerve the following proportion;

As the mean diameter : is to 100 (the diameter of the tabular circle) : : ſo is the laſt difference, (viz. $x$) to a verſed ſine in the table, page 182.

Then if the tabular ſegment, which ſtands againſt that verſed ſine, be multiplied into the content of the caſk; the product will ſhew the ullage; viz. what quantity of liquor is either in the caſk or drawn out.

## EXAMPLE.

Take a caſk, whoſe bung diameter is 31,5 inches; mean diameter 29,05; and content 98,71 ale gallons; then ſuppoſe there were 10,5 inches wet in it; it is required to find the wet and dry gallons.

Here 31,5—29,05=2,45; its half is 1,22. And 10,5—1,22=9,28. Then 29,05 : 100 : : 9,28 : 0,319= V. ſine; its ſegment is 0,2748, And 98,71×0,2748=27,12 the number of wet gallons.

Again, 31,5—10,5=21, the dry inches; and 21,—1,22=19,78. Then 29,05 : 100 : : 19,78 : 0,68; its ſegment is 0,7241. And 98,71×0,7241=71,48, the number of dry gallons.

Proof, 71,48×27,12=98.6, is nearly the content of the caſk.

CHAP.

## CHAP. II.

### *Of the practical methods of measuring timber, stone, &c.*

*Philo.* THESE branches of mensuration being of the utmost importance, I could wish, my dear Discipulus, that you would be very attentive to the instructions I shall give you, relative to these subjects, because they will be found highly useful on many occasions.

*Discip.* It is hardly possible for any person, desirous of understanding those branches of learning, to neglect so fair an opportunity of increasing his knowledge, and acquiring a system of instructive lessons, that cannot fail of being highly useful to him during his whole life. For my own part, I value your instructions at so high a rate, that I shall always think the day you were kind enough to take me under your care, as one of the most fortunate in my whole life.

*Philo.* It gives me great satisfaction to hear you are so fond of your learning: and give me leave to add, that the rules I shall lay down with regard to those practical branches of mensuration, will be so plain and easy to be understood, that you will find no difficulty in putting them in practice, whenever there shall be occasion.

## SECT. I.

### *Of Measuring Timber.*

In order to render this art more intelligible, I shall divide it into three branches. In the first I will shew the method of measuring square or equal sided timber: in the second, that of unequal sided timber: and in the third, I will explain the various methods used by practical measurers for finding the content if round timber, whether it be of the same diameter throughout, or tapering, as trees, &c.

### 1. *Of meaſuring ſquare timber.*

*Diſcip.* What am I to underſtand by ſquare timber?

*Philo.* By ſquare timber is to be underſtood, all ſuch pieces as are hewed, or ſawed into a true parallel-piped; or whoſe breadth and depth are equal to one another. And all theſe are meaſured by the following

### RULE.

Find the area of the baſe, (that is, multiply the breadth by the depth) and multiply that area or product by the length: the reſult of the laſt operation will be the content deſired.

### EXAMPLE I.

Suppoſe a piece of timber be one foot, nine inches in breadth, the ſame in depth, and 20 feet long: how many ſolid feet does it contain, and what will it come to at 1*s.* 9*d.* per foot?

*Decimal Solution.*

```
   1.75 Breadth
   1.75 Depth
  ------
    875
  1225
  175
  ------
  3.0625
      20 Length
  --------
  61.2500 Content.
```

As one foot : 1*s.* 9*d.*=1.75 : : 61.25 :

```
         61,25
          1,75
         ------
         30625
        42875
        6125
      ----------
   s. 107,1875
            12
      ----------
      d. 2,2500
             4
      ----------
      f. 1,00
```

Anſ. The content is 61 feet and a quarter, and the price 5*l.* 7*s.* 2*d.* $\frac{1}{4}$.

*By Croſs Multiplication.*

```
1—9
   1—9
---------
1—3—9
1—9—
---------
3—0—9
     20
---------
61—3—0
```

*Diſcip.* I am ſorry to interrupt you—but I have often heard workmen mention a *load* of timber; what do they mean by that term?

*Philo.* By a load of timber workmen and meaſurers mean 40 feet of rough or unhewn timber, and 50 feet of hewn timber; and this quantity is ſuppoſed to weigh a ton, or 20 hundred weight. The reaſon they give for this difference between a load of hewn or unhewn timber is, that the former is meaſured very near the truth; whereas the latter is ſo far from being ſo, that what is ſold for 40 is about 50 feet; conſequently the weight of each is nearly equal. So that upon the whole, it amounts to the ſame thing. Hence you may eaſily reduce the ſolid content of any piece or pieces of timber to loads by the following

RULES.

## RULES.

Divide the feet of hewn timber by 50; the quotient gives the loads.

Divide the feet in rough timber by 40; the quotient gives the loads.

There is, however, ſome difference with regard to the terms in the King's Yards. They call 40 feet of hewn timber a ton, and 50 feet a load.

## EXAMPLE.

Suppoſe a merchant has ſeven pieces of ſquare timber to diſpoſe of, at 30*s.* 6*d.* per load, of the following dimenſions, viz. No. 1, 2 feet, 6 inches the ſide of the ſquare, and 30 feet long: No. 2. 3 feet, 3 inches the ſide of the ſquare, and 40 feet long: No. 3. 2 feet 9 inches the ſide of the ſquare, and 25 feet long: No. 4. 4 feet 2 inches the ſide of the ſquare, and 18 feet 6 inches long: No. 5. 3 feet 9 inches the ſide of the ſquare, and 22 feet long: No. 6. 4 feet the ſide of the ſquare, and 18 feet long: and No. 7. 5 feet 3 inches the ſide of the ſquare, and 28 feet long. What is the ſolid content of the whole; and what will it amount to at the above price?

*Decimal Solution.*

NUMB. I.

```
  2.5
  2.5
 ----
  125
  50
 ----
 6.25
   30 Length
 ------
187.50 Content.
```

NUMB.

NUMB. II.

```
       3.25
       3.25
     ------
       1625
       650
      975
   --------
    10.5625
         40 Length
   --------
   422,5000 Content.
```

NUMB. III.

```
       2.75
       2.75
     ------
       1375
      1925
     550
   --------
     7.5625
         25 Length
   --------
     378125
    151250
   --------
   189.0625 Content.
```

NUMB. IV.

```
        4.166
        4.166
     --------
        24996
       24996
      4166
     16664
   ----------
     17,355556
          18,5
   ----------
      86777780
    138844448
    17355556
   ------------
   321,0777860 Content.
```

NUMB. V.

```
   3.75
   3.75
   ----
   1875
  2625
 1125
 -------
 14.0625
      22 Length
 -------
  281250
 281250
 --------
 309.3750 Content.
```

NUMB. VI.

```
   4
   4
  --
  16
  18 Length.
  --
 128
 16
 ---
 288 Content.
```

NUMB. VII.

```
   5.25
   5.25
   ----
   2625
  1050
 2625
 -------
 27.5625
      28
 -------
 2205000
 551250
 --------
 771.7500 Content.
```

Contents.

No. 1.——187.5
2.——422.5
3.——189.0625
4.——321.077786
5.——309.375
6.——288
7.——771.75

5'0)2489.26528(6 Content.

As 1 load : 1*l.* 10*s.* 6*d.* = 1.5025 : : 497.853|05 loads.

```
           1.5025
         2489265
         995706
       24892650
       497853
       748.0241325
                20
          .4826500
```

Anſw. The number of loads is 497.853; the price 748*l.*

As I have given the whole operation by decimals at large, I ſhall leave the ſolution by duodcimals, or croſs multiplication, as an exerciſe to yourſelf, fully perſuaded that you will find no difficulty in the taſk: and ſhall conclude the menſuration of ſquare timber, with propoſing the following queſtions, and giving their anſwers, without the arithmetical operations.

## QUESTION I.

How many ſolid feet are contained in a piece of timber, when the ſide of the ſquare is 2 feet 11 inches, and the length 37 feet?

Anſw. 314 feet, 9 inches, 1 twelfth of an inch.

QUES-

### QUESTION II.

Suppoſe a piece of timber, 1 foot 8 inches the ſide of the ſquare, and 29 feet 6 inches long; what is its ſolid content?

Anſ. 81 feet, 11 inches, 4 twelfths.

### QUESTION III.

If the ſide of the ſquare of a piece of timber be 3 feet 7 inches, and its length 18 feet; what is its ſolid content?

Anſ. 231 feet 1 inch and a half.

### QUESTION IV.

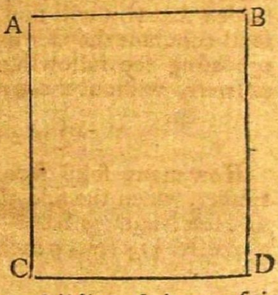
It is required to find the ſolidity of a piece of timber, the ſide of the ſquare being 1 foot 7 inches, and the length 31 feet 6 inches.

Anſ. 78 f. 11 in. 7. p.

### 2. *Of meaſuring unequal ſided Timber.*

*Diſcip.* What am I to underſtand by *unequal ſided* timber.

*Philo.* That the breadth of a piece of timber is greater than its depth, or its depth greater than its breadth; that is, when theſe two dimenſions are unequal.




Thus, if ABCD, repreſent the baſe of a piece of timber. and AC = BD be longer than AB = CD and theſe dimenſions are the ſame through the whole piece, it is ſaid to be unequal ſided.

*Diſcip.* I am greatly obliged to you for this explanation; and ſhall now be glad to know how I am to proceed, in order to find the ſolidity of pieces of timber of this kind.

*Philo.*

*Philo.* The whole difficulty of meaſuring theſe pieces by tables, conſiſts in reducing the baſe to a ſquare; that is, in finding the ſide of a ſquare, whoſe area is equal to that of a parallelogram, whoſe dimenſions are given: and this difficulty is eaſily removed by the following

## RULE.

Multiply the two ſides together, and extract the ſquare root out of the product: that root will be the ſide of the ſquare required.

But in order to find the ſolidity by arithmetical computation, there is not the leaſt neceſſity for this trouble: you have nothing more to do than to multiply the breadth by the depth, and that product by the length; for the laſt reſult will be the ſolidity required.

## EXAMPLE I.

What is the ſolid content of a piece of timber, whoſe breadth is 2 feet 6 inches, its depth 1 foot 9 inches, and its length 20 feet?

*Decimal Solution.*

```
 1.75  Depth
  2.5  Breadth.
 -----
  875
 350
 -----
4.375  Area of the baſe
   20  Length.
 -----
87.500 Solidity.
```

Anſ. 87.5, or 87 feet and a half.

## EXAMPLE II.

Suppoſe a piece of timber be 4 feet broad, 3 feet 6 inches deep, and 30 feet long, what is the ſolid content of that piece?

Decimal

*Decimal Solution.*

```
   3.5 Depth
     4 Breadth
  ----
  14.0
    30 Length.
  ----
   420
```

Anſ. 420 feet.

## EXAMPLE III.

If on meaſuring a piece of timber, I find it 2 feet 3 inches broad, 2 feet 9 inches deep, and 40 feet long; how many ſolid feet does the piece contain?

*Solution by Decimals.*

```
    2.25 Breadth
    2.75 Depth.
  ------
    1125
   1575
   450
  ------
  6.1875
      40
  ------
247.5000
```

Anſw. 247.5 feet.

*Diſcip.* You mentioned, in meaſuring unequal ſided timber by tables, it was neceſſary to multiply the dimenſions of the baſe together, and extract the ſquare root out of the product, in order to find the ſide of a ſquare, whoſe area ſhould be equal to the area of the parallelogram. I ſhould be glad to know what you mean by meaſuring timber by tables?

*Philo.* There are ſeveral treatiſes extant, by which you may readily find the ſolidity of any piece of timber, according to the practical method of meaſuring without the help of any arithmetical calculations, as I ſhall explain more fully, when I come to ſhew you the manner of meaſuring tapering timber, where I propoſe

to

to explain the method uſed in computing thoſe tables, and the uſe that ſhould be made of them. In the mean I ſhall purſue the method I have laid down at the beginning of this ſection.

### 3. *Of meaſuring round timber.*

Round timber is of two kinds, firſt, That where the two ends are equal to each other. Second, That where the two ends are not equal to each other, and therefore called tapering timber.

### 1. *Of meaſuring round timber, when both the ends are equal.*

Pieces of round timber, where both the ends are equal, are cylinders, and their ſolidity may be found by the following

### RULE.

Multiply the ſquare of one fourth of the circumference by the length, and the product is the ſolidity required.

### EXAMPLE I.

Let it be required to find the ſolid content of a piece of round timber, whoſe circumference is 4.496 feet, and its length 10 feet.

*Solution*

*Solution by Decimals.*

```
    1.124 one fourth of the circumference.
    1.124
   ------
     4496
    2248
   1124
  1124
 --------
 1.263376
       10
 ---------
12.633760
       12
 ---------
  7.80512
```

Anſ. 12.63376 feet; or 12 feet, 7 inches, and 8 tenths of an inch.

## EXAMPLE II.

What is the content of a piece of round timber, whoſe circumference is 8 feet 4 inches, and its length 15 feet; ſuppoſing its ends are equal?

```
      2.0833 one quarter of the circumf.
      2.0833
     -------
       62499
      62499
    166604
  41 6660
 ----------
 4.34013889
         15 Length.
 ----------
 2170069445
 434013889
 ----------
 65.10208335
```

Anſ. 65 feet. 1 inch, and 2 tenths of an inch.

As the above operation is compoſed of a great number of figures, the major part of which are of no real uſe, the work may be much ſhortened by inverting the multiplicator, as explained in page 25 of the introduction to this treatiſe.

*Discip.* I perceive there is no difficulty in finding the solidity of any piece of round timber, provided the dimensions are given; but I should be glad to know, what method is taken by practical measurers to find the circumference.

*Philo.* The method is very easy, being nothing more than this: They girt the piece round with a string, double the string twice, and measure it on a rule, by which means they obtain one quarter of the girt or circumference, which they consider as the side of a square, whose area is equal to the area of the circular base.

*Discip.* But is this method true?

*Philo.* No: it is very erroneous; for the fourth part of the circumference of a circle, is not equal to the side of a square, whose area is equal to the area of the circle. For if the circumference of a circle be 1, the side of a square of equal area to that circle, is 0.2821, (see page 29) whereas by the false method of the girt it is only 0.25.

*Discip.* Why then is it practised? why is not the true method used instead of the false?

*Philo.* It was first introduced by ignorance, and continued, because it is somewhat easier than the other, till custom fixed it so firmly, that all the demonstrations in the world will not be able to banish it from general practice. But the solidity of round timber may be found very near the truth, by the following

## RULE.

Multiply the square of the circumference of the piece by the length, and that product by 0,07958; the last result will be the solidity required.

## EXAMPLE I.

What is the solidity of a piece of timber, the fourth of whose circumference is 1 foot 6 inches, and its length 20 feet?

*Solu-*

*Solution by Decimals.*

```
      6 Circumference.
      6
     --
     36
     20 Length.
   ----
    720
 .07958
 ------
   5760
  3600
 6480
5040
--------
57.29760
      12
--------
3.57120
      12
--------
6.85440
```

Anſ. 57 feet, 3 inches, 6 parts.

*Solution according to the common Method.*

```
 1.5 one fourth of the girt.
 1.5
 ---
  75
 15
----
2.25
  20 Length.
----
45.00
```

Anſ. 45 feet.

Hence it appears, that 12 feet, 3½ inches of timber are loſt to the ſeller by uſing the common method.

## EXAMPLE II.

Required the ſolidity of a piece of timber, when the fourth of the girt is 2 feet, and the length 30 feet.

*Solution by the true method.*

```
        8 Circumference.
        8
       --
       64
       30 Length.
     ----
     1920
   .07958
   ------
    15360
    9600
  17280
 13440
---------
152.79360
```

Anſ. 152.7936 feet, or 152 feet 9 inches and a half.

*Solution by the common Method.*

```
  2
  2
 --
  4
 30
---
120 Feet, the anſwer.
```

The difference between theſe two ſolutions is 32 feet 9 inches and a half.

I ſhall add, my dear Diſcipulus, a ſolution to this queſtion, by the accurate method laid down in page 134, for finding the ſolid content of a cylinder.

The diameter of a circle, whoſe circumference is 8, will be 2.546, this being ſquared is 6.482116, which multiplied by .7854, (ſee page 28) is 5.0910429064, the area of the baſe; this area multiplied by the length gives the ſolidity required.

OPERATION.

OPERATION.

```
        2.546
        2.546
       ------
        15276
       10184
      12730
      5092
     --------
     6.482116
        .7854
     --------
     25928464
    32410580
   51856828
  45374712
  -----------
  5.0910429064
           30 Length.
  -------------
 152.731287 1920
```

Anſ. 152.73, &c. feet, not differing from the firſt ſolution one tenth of an inch; a ſufficient proof that the above rule is ſufficiently near the truth for common practice.

## 2. *Of meaſuring tapering timber.*

*Diſcip.* What am I to underſtand by tapering timber?

*Philo.* By tapering timber is meant, trees, &c. as they grow in the foreſts, or any pieces of timber, that are larger at one end than the other.

*Diſcip.* What method muſt be purſued in order to find the ſolid content of ſuch pieces of timber?

*Philo.* The method generally uſed by meaſurers is very eaſy and expeditious; it is this: They meaſure along the piece from one end to the other, for the length; and at the middle, they girt it round with a ſtring, double that ſtring twice, and meaſure the length of it thus doubled, and call this the fourth of the girt, and ſuppoſe it equal to the ſide of the ſquare, which the piece will hold throughout. In conſequence of this, they ſquare the fourth part of the girt, that is,



they

they multiply it by itself; and that product multiplied by the length of the piece, gives the content.

But if the tree has its bark on, they make an allowance for it; deducting it from the fourth part of the girt before they multiply it by itself.

*Discip.* What are the general allowances made for the bark?

*Philo.* There is not any fixed proportion for this allowance, that is general in all places; but the following are commonly used. For oak one tenth, or one twelfth part of the whole girt is deducted for the bark; in elm, beech, ash, &c. it is something less. Some measurers insist upon one inch, out of the fourth part of the girt. That is, if the quarter of the girt be 10 inches, they call it nine, and with that proceed to measure the tree. When the allowance for the bark is settled, the tree is measured by the following

## RULE.

From the girt, or quarter of the girt, according to the nature and thickness of the bark, deduct the allowance, and with the remainder find the content of the tree, as mentioned above.

## EXAMPLE I.

Suppose it were required to find the content of an old thick-barked tree, whose length is 22 feet, and the quarter of the girt 2 feet 10 inches; the allowance for the bark being 1 inch out of the fourth of the girt.

Operation

*Operation by Decimals.*

```
  2.8333
   .0833 Allowance for bark.
 -------
  2.7500
  2.75
 -------
   1375
  1925
 550
 -------
 7.5625
     22 Length.
 -------
 151250
151250
 -------
166.3750
     12
 -------
  4.500
```

Anſ. 166 feet, 4½ inches.

*By croſs Multiplication.*

```
 2—10
     1 Allowance for bark.
 -------
 2—-9
    2—9
 -------
 2—-0—9
 5—-6
 -------
 7—-6—9
     22 Length.
 -------
 26—4—6
 14
 -------
 166--4 6
```

Anſ. 166 feet, 4 inches, and 6 twelfths, or 4½ inches, the ſame as before.

EXAMPLE II.

What is the content of a tree, whoſe length is 25 feet, and the quarter of the girt 3 feet 10 inches, the

 allow-

allowance for bark being one twelfth of the circumference.

*Operation by Decimals.*

```
12)3.8333|.3194
   3.6
   ---
    23
    12
    ---
    113
    108
    ---
     53
     48
     ---
     .5

      3.8333
       3194
      ------
      3.5139
      3.5139 reduced quarter of the girt.
      ------
      316251
     105417
     35139
   175695
   105417
  -----------
  12.34749321
           25 Length.
  -----------
  6173746605
 2469498642
 ------------
 308.68733025
        12
 ------------
   8.244
      12
 ------------
   2.928
```

Anf. 308 feet, 8 inches, 2 parts and nine tenths.

By

*By cross Multiplication.*

```
        12|3—10
        ————————
           0—-3—10
       ——————————————
From  3—10
Take        3—10
       ——————————————
Rem.  3—-6—-2 Reduced ¼ girt.
          3—-6—2
       ——————————————
               7—0—4
       1—-9—-1—0
      10—-6—-6
       ——————————————
      12—-4—-2—0
             25 Length.
       ——————————————
      68—-8—-2
      24
       ——————————————
     308---8---2
```

Ans. 308 feet, 8 inches, 2 twelfths; nearly the same as before.

## Example III.

A timber merchant brought three pieces of timber of the following dimensions, after the bark is allowed for, viz. No. I. 20 feet long, and 2 feet 3 inches the quarter of the girt; No. II. 30 feet long, and 1 foot 9 inches the quarter of the girt; and No. III. 27 feet long, and 2 feet 6 inches the quarter of the girt, at 1s. per foot; what will the whole amount to?

*Operation by Decimals.*

NUMB. I.

```
        2.25
        2.25
       -----
        1125
        450
       450
      ------
      5.0625
          20
   ---------
F. 101.2500
         12
   ---------
 I. 3.00
```

NUMB. II.

```
        1.75
        1.75
      ------
         875
       1225
       175
     -------
      3.0625
          30
    --------
F. 91.8750
         12
    --------
I. 10.500
```

NUMB. III.

```
        2.5
        2.5
      -----
        125
        50
      -----
       6.25
         27
      -----
       4375
      1250
   --------
F. 168.75
        12
   --------
I. 9.00
```

F. I.

F. I.

No. 1.——101.25
2.——91.875
3.——168.75

361.875

As 1 foot : 1s.(=.05) : : 361.875
.05

£. 18.09375
20

s. 1.87500
12

d. 10.500
4

f. 2.0

Anſ. The content of the whole is 361, feet, 10½ inches; and the price 18*l.* 1*s.* 10*d.* ½.

*By Croſs Multiplication.*

Numb. I.

2--3 Quarter girt.
2—3

6—9
4—6

5—0—9
20 Length.

101—3—0

Numb. II.

1—9 Quarter girt.
1--9

1--3--9
1--9

3--0--9
30 Length.

91-10--6

NUMB. III.

```
 2--6 Quarter girt.
   2--6
 -------
 1--3--0
   5--0
 -------
 6--3--0
     27
 -------
  48--9
   12
 -------
 168--9
```

| | F. | I. |
|---|---|---|
| No. 1. | 101 | 3 |
| 2. | 91 | 10½ |
| 3. | 168 | 9 |
| | 361 | 10½ |

Anſw. 361 feet 10½ inches; the ſame as before.

## EXAMPLE IV.

A merchant has bought a fall of timber, conſiſting of ten trees, whoſe dimenſions, when the bark is allowed for, are as follows:

| Quarter girt. | F. I. | Length. |
|---|---|---|
| No. 1. | 0--9 | 10 |
| 2. | 0-10½ | 14 |
| 3. | 1--3 | 11 |
| 4. | 1--8½ | 16 |
| 5. | 1--4 | 24 |
| 6. | 1--4 | 23 |
| 7. | 3--0 | 18 |
| 8. | 2--2 | 28 |
| 9. | 2--7 | 17 |
| 10. | 2--11 | 30 |

What will the whole amount to at 9*d*. ½ per foot?

*Operation*

*Operation by Decimals.*

NUMB. I.

```
    ·75 Quarter girt.
    ·75
   ----
    375
   525
  -----
  ·5625
     10 Length.
 -------
 5.6250 Content.
```

NUMB. II.

```
     .875 Quarter girt.
     ·875
    -----
     4375
    6125
   7000
  -------
  ·765625
       14 Length.
 ---------
  3062500
  765625
 ---------
 10.718750 Content.
```

NUMB. III.

```
    1.25 Quarter girt.
    1.25
   -----
     625
    250
   125
  ------
  1.5625
      11 Length.
 --------
 17.1875 Content.
```

NUMB.

NUMB. IV.

```
   1.708  Quarter girt.
   1.708
   -----
   13664
  119560
  1708
---------
2.917264
      16  Length.
---------
18503584
2917264
---------
46.676224 Content.
```

NUMB. V.

```
    1.3333  Quarter girt.
    1.3333
    ------
     39999
    39999
   39999
  39999
 13333
----------
1.77768789
        24  Length.
----------
711075156
355537578
-----------
42.66450736 Content.
```

NUMB. VI.

```
 1.3333  Quarter girt.
 3333.1  Multiplier inverted.
 ------
 13333
  3999
   399
    39
     3
 ------
 1.7776
     23  Length.
 ------
  53328
 35552
 -------
40.8848  Content.
```

NUMB.

NUMB. VII.

```
  3 Quarter girt.
  3
 ---
  9
 18 Length.
---
162 Content.
```

NUMB. VIII.

```
   2.16666 Quarter girt.
   66661.2 Multiplier inverted.
  ---------
   433333
    21666
    12099
     1299
      129
       12
  ---------
   4.69131
        28 Length.
  ---------
  3753048
  938262
  ---------
131.35668 Content.
```

NUMB. IX.

```
  2.58333 Quarter girt.
  33385.2 Multiplier inverted.
 ---------
  516666
  129166
   20666
     774
      75
       7
 ---------
 6.67357
      17 Length.
 ---------
 4671499
 667357
 ---------
113.45069
```

NUMB.

NUMB. X.

2.91666 Quarter girt.
66619 2 Multiplier inverted.

583333
262499
29 6
1749
174
17

8.50792
30 Length.

255.23760 Content.

No. 1.——5.625
2.——10.71875
3.——17.1875
4.——46.67622
5.—— 42.6945
6.——40.8848
7.--- 162.
8.--- 131.3568
9.--- 113.45069
10.--- 255.2376

825.79186

As 1 foot : to *9d.* ½ (=.03958) : : 825.79183
85930 inverted.

24773
7432
412
65

£. 32.684
20

*s.* 13.680
12

Anf. 32*l.* 13*s.* 8*d.* *d.* 8.16

By

*By cross Multiplication.*

NUMB. I.

0--9 Quarter girt.
9--9
6--9
10 Length.
5--7--6 Content.

NUMB. II.

0-10--6 Quarter girt.
0-10--6
0--5--3--0
8--9--0
9--2--3
14 Length.
10--8--7--6 Content.

NUMB. III.

1--3 Quarter girt.
1--3
3--9
1--3
1--6--9
11 Length.
17--2--3

NUMB. IV.

1--8--6 Quarter girt.
1--8--6
10-3--0
1--1--8--0
1--8--6
2-11--0--3--0
16 Length.
46--8--4--0 Content.

NUMB. V.

1--4 Quarter girt.
1--4
5--4
1--4
1--9--4
24 Length.
42--8--0 Content.

NUMB. VI.

1--4 Quarter girt.
1--4
5--4
1--4
1--9--4
23 Length.
40-10--8 Content.

NUMB. VII.

3--0 Quarter girt.
3--0
9
18 Length.
162 Content.

NUMB. VIII.

2--2
2--2
4--4
4--4
4--8--4
28 Length.
131--5--4 Content.

NUMB. IX.

2—7 Quarter girt.

2--7

1--6--1

5--2

6--8--1

17

113--5--5 Content.

NUMB. X.

2--11 Quarter girt.

2--11

2---8---1

5--10

8---6---1

30 Length.

255---2---6 Content.

No. 1.——5——7——6
2.——10——8——7——6
3.——17——2——3
4.——46——8——4
5.——42——8
6.——40——10——8
7.——162.
8.——131——5——4
9.——113——5——5
10.——255——2——6
Total 825——10——7--6

Such is the method generally uſed in meaſuring timber; but in order to prevent miſtakes, ſeveral authors have calculated tables, whereby the content of any piece of timber is eaſily known, by having the length and the quarter of the girt given. Nor is it at all difficult, though ſomething tedious to calculate ſuch tables for your own uſe.

*Diſcip.* I ſhould be very glad to know the method taken in forming tables of this kind: I have ſeen tables of various kinds, in books I have occaſionally read on other ſubjects; but always thought they could not

not be composed without the utmost labour and difficulty.

*Philo.* Some tables are indeed sufficiently difficult and tedious; but the generality, especially those for measuring timber, are easily computed. I will give you an instance or two of this, by computing part of a table of that kind. But it will be previously necessary to observe, that a table sufficient to answer the purposes of measuring timber, must be computed from six inches to 54 inches the side of the square, and from one fourth of a foot to 45 feet in length.

This being premised, let it be required to compute a table for measuring a piece of timber, whose side of the square is seven inches, but of any length less than 26 feet.

In order to find the content of one foot in length, multiply the seven inches by seven inches, and the result will be four inches and one twelfth, thus:

```
7--0
  7--0
------
4--1
```

This will be the first line in the table; the second line will be the first doubled, or added to itself; the third the first and second added together; the fourth, the third, and first added together: and thus, by a continual addition of the first line, the table will be readily constructed without any farther difficulty. See the whole work.

| F. long. | F. | I. | P. |
|---|---|---|---|
| 1 | 0 | 4 | 1 |
| | | 4 | 1 |
| 2 | 0 | 8 | 2 |
| | | 4 | 1 |
| 3 | 1 | 0 | 3 |
| | | 4 | 1 |
| 4 | 1 | 4 | 4 |
| | | 4 | 1 |
| 5 | 1 | 8 | 5 |
| | | 4 | 1 |

F. long

| F. long. | F. | I. | P. |
|---|---|---|---|
| 6 | 2 | 0 | 6 |
| | | 4 | 1 |
| 7 | 2 | 4 | 7 |
| | | 4 | 1 |
| 8 | 2 | 8 | 8 |
| | | 4 | 1 |
| 9 | 3 | 0 | 9 |

It will, perhaps, be unneceſſary to obſerve, that the figures 4--1, that ſtand between the lines are to be omitted, when the table is finiſhed; they are only placed there to ſhew the method of computing the contents that are ſet againſt their reſpective lengths. When they are omitted, the numbers will ſtand as below.

A Table ſhewing the ſolid content of any piece of timber, ſeven inches the ſide of the ſquare, and not exceeding 26 feet in length.

| F. long. | F. | I. | P. |
|---|---|---|---|
| 1 | 0 | 4 | 1 |
| 2 | 0 | 8 | 2 |
| 3 | 1 | 0 | 3 |
| 4 | 1 | 4 | 4 |
| 5 | 1 | 8 | 5 |
| 6 | 2 | 0 | 6 |
| 7 | 2 | 4 | 7 |
| 8 | 2 | 8 | 8 |
| 9 | 3 | 0 | 9 |
| 10 | 3 | 4 | 10 |
| 11 | 3 | 8 | 11 |
| 12 | 4 | 1 | 0 |
| 13 | 4 | 5 | 1 |
| 14 | 4 | 9 | 2 |
| 15 | 5 | 1 | 3 |
| 16 | 5 | 5 | 4 |
| 17 | 5 | 9 | 5 |
| 18 | 6 | 1 | 6 |
| 19 | 6 | 5 | 7 |
| 20 | 6 | 9 | 8 |

F. long.

| F. long. | F. | I. | P. |
|---|---|---|---|
| 21 | 7 | 1 | 9 |
| 22 | 7 | 5 | 10 |
| 23 | 7 | 9 | 11 |
| 24 | 8 | 2 | 0 |
| 25 | 8 | 6 | 1 |
| 26 | 8 | 8 | 2 |

If you calculate a ſet of tables in this manner, for every quarter of an inch the ſide of the ſquare, and extend the length to 45 feet, you will have a very uſeful compendium for meaſuring timber with accuracy and diſpatch. There are indeed ſeveral books extant filled with tables of this kind: But I would not have you depend implicitly upon any tables, till you have thoroughly examined and corrected them; as there are errors even in the beſt.

With regard to the quarter, half, or three quarters of a foot, happening often in the length, it will be ſufficient to calculate the contents of theſe, and ſet them down at the bottom of the table. Theſe contents are eaſily found, if you conſider that the one quarter of a foot is one fourth of the firſt line, &c. and therefore the contents of theſe fractions are :

| | F. I. P. S. |
|---|---|
| $\frac{1}{4}$ | 0--1--0--3. |
| $\frac{1}{2}$ | 0--2--0--6. |
| $\frac{3}{4}$ | 0--3--0--9. |

Theſe contents, when they happen in the length, are to be added to the contents found in the table, and the ſum will be the area required.

## EXAMPLE.

What is the ſolid content of a piece of timber, whoſe ſide of the ſquare, or quarter girt, is ſeven inches, and the length 23 feet nine inches, or 23 feet and three quarters.

| | F. I. P. |
|---|---|
| The content of 23 feet | 7--9--11 |
| $\frac{3}{4}$ | 0--3---0--9 |
| | 8--0--11--9 |
| | Anſ. |

Anſ. 8 feet, 0 inches, 11 parts, 9 ſeconds.

In order to render the method of calculating, and conſequently of examining tables as eaſy as poſſible, I ſhall give another inſtance, where the firſt line has ſecond and third parts in it, as the method of proceeding is ſomething different from that of the former.

Let it be required to compute a table for meaſuring timber, whoſe quarter girt, or ſide of the ſquare, is eight inches and a quarter.

```
   8--3
     8--3
  2--0--9
 5--6--0
 5--8--0--9  firſt line.
```

| F. long. | F. | I. | P. | S. | T. |
|---|---|---|---|---|---|
| 1 | 0 | 5 | 8 | 0 | 9 |
| | 0 | 5 | 8 | 0 | 9 |
| 2 | 0 | 11 | 4 | 1 | 6 |
| | 0 | 5 | 8 | 0 | 9 |
| 3 | 1 | 5 | 0 | 2 | 3 |
| | 0 | 5 | 8 | 0 | 9 |
| 4 | 1 | 10 | 8 | 3 | 0 |
| | 0 | 5 | 8 | 0 | 9 |
| 5 | 2 | 4 | 4 | 3 | 9 |
| | 0 | 5 | 8 | 0 | 9 |
| 6 | 2 | 10 | 0 | 4 | 6 |
| | 0 | 5 | 8 | 0 | 9 |
| 7 | 3 | 3 | 8 | 5 | 3 |
| | 0 | 5 | 8 | 0 | 9 |
| 8 | 3 | 9 | 4 | 6 | 0 |
| | 0 | 5 | 8 | 0 | 9 |
| 9 | 4 | 3 | 0 | 6 | 9 |

In this manner the above table is computed, and when the intermediate lines, together with the ſecond and third parts are omitted, the contents will ſtand in the following manner:

| F. long. | F. | I. | P. |
|---|---|---|---|
| 1 | 0 | 5 | 8 |
| 2 | 0 | 11 | 4 |
| 3 | 1 | 5 | 0 |
| 4 | 1 | 10 | 8 |
| 5 | 2 | 4 | 4 |
| 6 | 2 | 10 | 0 |
| 7 | 3 | 3 | 8 |
| 8 | 3 | 9 | 4 |
| 9 | 4 | 3 | 0 |
| 10 | 4 | 8 | 8 |
| 11 | 5 | 2 | 4 |
| 12 | 5 | 8 | 0 |
| 13 | 6 | 1 | 8 |
| 14 | 6 | 7 | 4 |
| 15 | 7 | 1 | 0 |
| 16 | 7 | 6 | 9 |
| 17 | 8 | 0 | 5 |
| 18 | 8 | 6 | 1 |
| 19 | 8 | 11 | 9 |
| 20 | 9 | 5 | 5 |
| 21 | 9 | 11 | 1 |
| 22 | 10 | 4 | 9 |
| 23 | 10 | 10 | 5 |
| 24 | 11 | 4 | 1 |
| 25 | 11 | 9 | 9 |
| 26 | 12 | 3 | 5 |
| 27 | 12 | 9 | 1 |
| 28 | 13 | 2 | 9 |
| 29 | 13 | 8 | 5 |

The Quarters are:

| | F. | I. | P. | S. |
|---|---|---|---|---|
| $\frac{1}{4}$ | 0 | 1 | 5 | 0 |
| $\frac{1}{2}$ | 0 | 2 | 10 | 0 |
| $\frac{3}{4}$ | 0 | 4 | 3 | 0 |

By this table you may very readily find the ſolid content of any piece of timber, whoſe ſide of the ſquare, or quarter girt, is $8\frac{1}{4}$ inches, and the length not exceeding 29 feet.



EXAMPLE.

EXAMPLE.

What is the solid content of a piece of timber, whose quarter girt is 8 ¼ inches, and its length 24 feet and a half?

| | F. | I. | P. |
|---|---|---|---|
| Content of 24 feet. | 11 | 4 | 1 |
| of ½ | 0 | 2 | 10 |
| | 11 | 6 | 11 |

Anſ. 11 feet, 6 inches, 11 parts, or ¾ of an inch.

*Diſcip.* I was unwilling to interrupt you, till you had finiſhed the ſolution, but ſhould now be glad to know why the ſolid content of 3, 6, and 9 inches only, are computed, and all the intermediate inches omitted.

*Philo.* The reaſon for their being omitted is, the ſmall increaſe they would add to the content of the piece of timber: Meaſurers, indeed, ſeldom take notice of any other denominations in the length of a piece, than feet and quarters. But this is a defect that ſhould be ſupplied in large pieces of timber, eſpecially when the wood is of a conſiderable price, ſuch as mahogany, &c. And I would recommend to you, in forming a ſet of tables, to remember, when the ſide of the ſquare is more than 12 inches, or one foot, to place at the bottom, the contents of the ſeveral inches; it will only add nine lines to each table, and, by that ſmall increaſe, you may compute the ſolidity to ſufficient accuracy. Let us, for inſtance, ſee the difference between the contents of a large piece of timber, when the intermediate inches are taken in, and when omitted in the computation.

EXAMPLE.

What will be the contents of two pieces of timber, one 3 feet 4 inches the ſide of the ſquare, and 20 feet 6 inches long; and the other 3 feet 4 inches the ſide of the ſquare, and 20 feet 8 inches long?

## OPERATION.

First piece 3——4 Side of the square.
3——4

1——1——4
10——0

11——1——4 Area of the base.
20——6 Length.

5——6——8——0
221---11——8

227——6——4——0

Second piece 3——4
3——4

1——1——4
10——0

11——1——4
20——8 Length.

7——4——10——8
221---11——4

229——4——2——8

| | F. | I. | P. | S. |
|---|---|---|---|---|
| The content of the 2d is | 229 | 4 | 2 | 8 |
| Ditto —— of the 1st | 227 | 6 | 4 | 0 |
| | .. 1 | 9 | 10 | 8 |

The difference is 1 foot, 9 inches, 10 parts, 8 second parts.

If you are desirous of computing decimal tables, it may be easily done; and they will have this advantage, that the content may be found to any degree of accuracy you may think necessary. One instance will be sufficient to shew the method of computing decimal tables for measuring timber.

## EXAMPLE.

Let it be required to compute a decimal table, for finding the solid content of any piece of timber, whose side of the square is 18 inches, or 1 foot 6 inches?

1.5

1.5 Side of the square.
1.5
———
75
15
———
2.25 First line.

1——2.25
2.25
———
2——4.20
2.25
———
3——6.75
2.25
———
4——9.00
2.25
———
5——12.25
2.25
———
6——14.50

In this manner the table may be easily constructed, and the numbers will stand as in the margin.

| | |
|---|---|
| 1 | 2.25 |
| 2 | 4.5 |
| 3 | 6.75 |
| 4 | 9. |
| 5 | 11.25 |
| 6 | 13.5 |
| 7 | 15.75 |
| 8 | 18 |
| 9 | 20.5 |
| 10 | 22.25 |
| 11 | 24.75 |
| 12 | 27 |
| 13 | 29.5 |
| 14 | 31.25 |
| 15 | 33.75 |
| 16 | 36 |
| 17 | 38.5 |
| 18 | 40.25 |
| 19 | 42.75 |
| 20 | 45 |
| 21 | 47.5 |
| 22 | 49.25 |
| 23 | 51.75 |
| 24 | 54 |
| 25 | 56.5 |
| 26 | 58.25 |
| 27 | 60.75 |
| 28 | 63 |
| 29 | 65.5 |
| 30 | 67.25 |
| 31 | 69.75 |
| 32 | 72 |

The uſe of this table is the ſame with the former, only the content here is given in feet and decimals, inſtead of feet, inches, and parts.

## EXAMPLE.

What is the content of a piece of timber, whoſe ſide of the ſquare is 1 foot 6 inches, and the length 19 feet.

Anſ. 42.75, or 42 feet, 9 inches.

What has been obſerved will be abundantly ſufficient to explain the nature and uſe of theſe tables; and I ſhall only obſerve, with regard to their conſtruction, that the lines forming a ſeries of numbers in arithmetical progreſſion, whoſe difference is the firſt line, there will often be no occaſion for the continued addition; but the numbers may be ſet down with the greateſt facility, without any calculation. The decimal table affords us a plain example of this kind, for the decimals are nothing more than an arithmetical progreſſion, increaſing by .25, or one quarter of a foot. The ſeries therefore is .25, .50 (or .5) .75, 0. And theſe will be continually repeated from the beginning of the table to the end, be its length what it will. The line of feet is alſo a ſeries of numbers in arithmetical progreſſion, whoſe common difference is 2, except where there is only a cypher, or blank, in the decimal ſeries, when the common difference will be 3. Conſequently the numbers are only 2, 4, 6, 9, 11, 13, 15, 18, and may be readily ſet down without any farther trouble, than that of adding three inſtead of two to every fourth number.

*Diſcip.* I am exceedingly obliged to you for the trouble you have taken in explaining theſe uſeful parts of menſuration in ſo conſpicuous a manner, but ſhould be glad to know the beſt method of finding the amount of any piece of timber, when the content is found by croſs multiplication. You have given a ſufficient number of inſtances by decimals; but as the other is often practiſed, and, I think, in ſome inſtances, with conſiderable advantage, I ſhould

often use it, provided I knew how to find the value, or what the piece will amount to at a certain rate per foot.

*Philo.* I am always pleased to satisfy any doubts that may perplex your mind, or remove any difficulties, that may obstruct your progress. I intended to give one instance of this kind before I finished this branch of mensuration: but as I perceive it is a difficulty of more consequence than I imagined, and that the removing this difficulty will give you pleasure, I shall be very explicit in this particular, and do not doubt of giving you full satisfaction.

In order to this, it will be necessary to consider the nature of the fractions into which the foot, &c. is divided; namely, twelfths. That is, the foot is divided into twelve equal parts called inches; the inches into twelve equal parts called the twelfth of an inch, or simply, parts; and these are again divided into other twelve equal parts called seconds, &c. Consequently the inches are twelfths of a foot, every part the 144th of a foot (=12×12) and every second the 1728th part (12×12×12).

The respective value of these divisions would be very readily found, if the denominations of money decreased in the same proportion: But this is not the case; there is only one denomination that has this proportion, viz. Shillings and pence. When, therefore, the price of timber is 1*s.* per foot, every inch will be a penny, and every part the twelfth of a penny, or one third of a farthing; consequently three of these parts are equal to a farthing. Hence it follows, that when the price of timber is any number of shillings per foot, the value is also very easily found; for if the whole content be multiplied by the number of shillings, the result will be the number of shillings. But when this does not happen, the value of the whole must be found, by taking the aliquot parts. A few examples will render the whole very intelligible.

## EXAMPLE I.

What will a piece of timber, 1 foot, 6 inches the side of the square, and 20 feet long, amount to at 1*s.* per foot?

### OPERATION.

```
1——6
      1——6
-----------
      9——0
1——6
-----------
2——3——0
        20
-----------
45——0——0 Content.
```

Ans. 45 shillings, or 2*l.* 5*s.*

## EXAMPLE II.

What will be the value of a piece of timber 25 feet long, and 1 foot 4 inches the side of the square, at 1*s.* per foot?

### OPERATION.

```
1——4
      1——4
-----------
      5—— 4
1——4
-----------
1——9——4
        25
-----------
44——5——4
```

| | £. | s. | d. |
|---|---|---|---|
| The 44 feet | 2 | 4 | 0 |
| The 5 inches | 0 | 0 | 5 |
| The 4 twelfths | 0 | 0 | 0¼ |
| | 2 | 4 | 5¼ |

Ans. 2*l.* 4*s.* 5d. ¼.

### EXAMPLE III.

A gentleman sold three pieces of timber of the following dimensions, viz.

| | F. I. | |
|---|---|---|
| No. 1.——— | 2--5 | Side of the square 25 feet long. |
| 2.——— | 1--9 | 27 |
| 3.——— | 1-10 | 40 |

What will the whole amount to at 2*s*. per foot?

### OPERATION.

| | F. | I. | P. |
|---|---|---|---|
| Content of No. 1.——— | 146 | 0 | 1 |
| 2.——— | 82 | 8 | 3 |
| 3.——— | 134 | 5 | 4 |
| Whole content | 363 | 1 | 8 |
| | £. | *s*. | *d*. |
| The 363 feet ——— | 36 | 6 | 0 |
| The 1 inch ——— | 0 | 0 | 2 |
| The 8 twelfths ——— | 0 | 0 | 1½ |
| | £36 | 6 | 3½ |

Ans. 36*l*. 6*s*. 3*d*. ½.

### EXAMPLE IV.

What must a timber-merchant pay for three pieces of timber, of the following dimensions, viz.

| | F. | S. | |
|---|---|---|---|
| No. 1.——— | 2 | 1 | Side square 20 feet long. |
| 2.——— | 3 | 2 | 30 |
| 3.——— | 1---11 | | 27 |

at 1*s*. 6*d*. per foot?

| | | | |
|---|---|---|---|
| Content of No. 1.——— | 86 | 9 | 8 |
| 2. ----- | 300 | 10 | 0 |
| 3.——— | 99 | 2 | 3 |
| Content of the whole | 486 | 9 | 11 |

| | | | |
|---|---|---|---|
| The 486 feet at 1*s*. per foot is | 24 | 6 | 0 |
| Ditto, at 6*d*. | 12 | 3 | 0 |
| The 9 inches at 1*s*. per foot | 0 | 0 | 9 |
| Ditto, at 6*d*. | 0 | 0 | $4\frac{1}{2}$ |
| The 11 twelfths | 0 | 0 | $1\frac{1}{2}$ |
| | £36 | 10 | 3 |

Anſ. 36*l*. 10*s*. 3*d*.

I have been ſo very explicit in theſe ſolutions, that I think any more will be unneceſſary, as the manner of proceeding in all is the ſame, the different price per foot making the only difference in the operation.

*Diſcip.* I am extremely obliged to you, Sir; you have removed all my difficulties; I can now find the value as eaſily as the content of any piece of timber.

*Of meaſuring ſtone, &c.*

*Philo.* Though I have thought proper to divide the menſuration of ſtone, &c. from that of timber, yet they are actually the ſame, and performed by the ſame rules, except in taking the dimenſions, which is done with more accuracy in ſtone, becauſe the value is generally much greater.

*Diſcip.* What are the methods uſed in taking the dimenſions of ſtone.

*Philo.* They meaſure the length, breadth, and thickneſs with a rule, but with ſuch accuracy, that even the twelfth of an inch is ſet down, in meaſuring a block of marble: in common ſtone they are not ſo accurate. When the dimenſions are taken, the ſolidity is found as before by the following

RULE.

Multiply the breadth into the depth or thickneſs, and that product by the length; the reſult is the ſolidity required.

EXAMPLE.

## EXAMPLE I.

What is the ſolid Content of a piece of marble, whoſe breadth is 4 feet, 8 inches, and a half, its depth 3 feet 4 inches, and its length 8 feet.

*Solution by Decimals.*

```
Breadth 4.7083
Depth   3.3333
       ---------
          141249
         141249
        141249
       141249
      141249
      ------------
      15.69417639
                8 Length.
   ---------------
F. 125.55341112
           12
   ---------
   I. 6.6408
           12
   ---------
   P. 7.6896
           12
   ---------
   S. 8.2752
```

Anſ. 125 F. 6 I. 7 P. 8 S.

*By Croſs Multiplication.*

```
   4——8——6
         3——4
   ----------------
   1——6---10    0
  14    1    6
  ---------------
  15    8    4
             8 Length.
  ---------------
  125   6    8 Content.
```

It will be needleſs to give any more inſtances of this kind, the operations being the very ſame with thoſe

already given for finding the solid content of square and unequal sided timber.

## SECT. II.

*Of the best methods of taking the dimensions of timber, in order to avoid the gross errors that attend the common practice of measurers.*

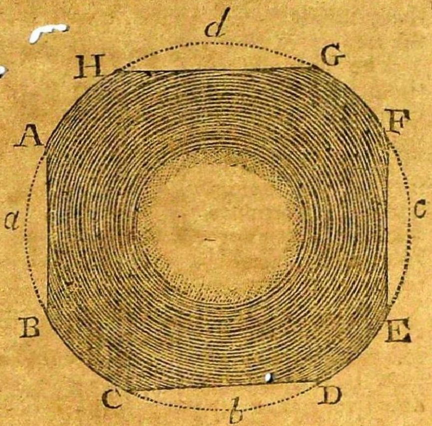
*Discip.* When you laid down the rule, page 214, I supposed, that if the method there pointed out was followed, the solution would be sufficiently accurate; can there then be any necessity for other rules and precepts?

*Philo.* The rule you mentioned is sufficiently accurate for finding the solidity of timber, whose ends are equal; but as this is rarely the case, other methods must be pursued, if the practitioner be desirous of avoiding the gross errors attending the common practice.

It is the custom in the King's yards, with regard to oak, elm, and beech, to have it squared by the merchant before it is received by the purveyors; and according to their contract, the sum of the breadth of the slabs taken off, are not to be less than twice the sum of the wanes; if they should be less the King's measurers hew the opposite sides till the dimensions are reduced to the terms of the contract; but they generally give the turn of advantage to the merchant. When the sides of the timber are thus square, the dimensions are taken with a pair of large callipers; and the different extents between the points of the instrument, being measured on a scale of inches, give the dimensions which they use instead of the girt.

*Discip.* In describing the method used for taking the dimensions of timber in the King's yards, you said, that the sum of the breadth of the *slabs* taken off, must not be less than twice the sum of the *wanes*. But as I have not an adequate idea of these terms, I should be glad if you would explain them to me.

Philo.




*Philo.* With all my heart Discipulus. The figure I have drawn, will, I dare say, be abundantly sufficient for the purpose. Let therefore, the outer circle A*d*GF*e*ED *b* CB *a* represent the end of a piece of timber; then will B*a*A, H*d*G, F*c* E, D*b*C, represent the *slabs* that must be taken off, and AH, GF, ED, CB, represent the *wanes*, or parts of the external circumference of the tree that are suffered to remain.

Now if the right lines AB, CD, EF, GH, are when added together, equal to twice the sum of the lengths of the arches or wanes AH, GF, ED, CB, the tree is properly prepared for the King's measurers; but if not, they hew the sides till they make them equal.

*Discip.* You have indeed removed the difficulty. I shall be at no loss in knowing when timber is, and when it is not properly prepared for being measured in the King's yards.

*Philo.* It gives me pleasure whenever my meaning is thoroughly understood; and I shall now proceed to lay down some useful rules, which you will do well to observe whenever you are called upon to measure timber.

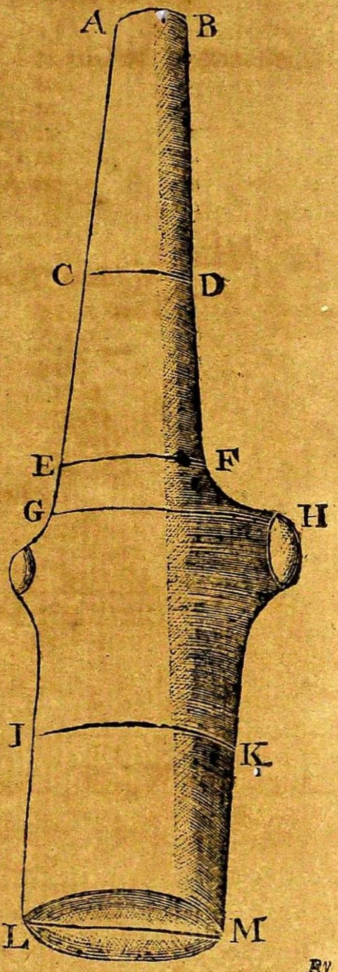
## RULE I.

Gird the tree in as many places as you conveniently can, add these girts together, and divide the sum by

their number; the quotient will be the mean girt of the tree.

The usefulness of this rule will sufficiently appear, if the figure I have drawn be attentively considered. It represents a tree in a form frequently met with, and which, if measured by the common method, will be sold at a very considerable loss to the seller. I have laid the figure down very accurately, from a scale, and the dimensions of it are these: the length is 31.7 feet, or 31 feet 8 inches, 4 parts, 3 seconds nearly. A quarter of the girt at LM=IK,=GH, is 1 foot 10 inches at EF, the middle of the tree, 1 foot 2 inches at CD, 1 foot 0, and at AB 7 inches.

Let it therefore be required to find the solid content of this tree, both by the preceding rule, and also by the common method of measuring.




By

*By the preceding Rule.*

| | | F. | I. |
|---|---|---|---|
| One-fourth of the Girt at | LM | 1 | 10 |
| | at IK | 1 | 10 |
| | at GH | 1 | 10 |
| | at EF | 1 | 2 |
| | at CD | 1 | 0 |
| | at AB | 0 | 7 |
| | | 8 | 3 |

6)8—3

1—4—6 Mean Quarter of the Girt=

```
Then 1.375
     1.375
     -----
      6875
     9625
    4125
   1375
  --------
  1.890625
      31.7 Length.
  --------
  13234375
  1890625
 5671875
 ----------
 59.9328125 Content.
F. 59.9328125
          12
   ----------
I. 11.193
       12
   ------
   2.316
```

Anſ. 59 feet, 11 inches, 2 twelfths.

*By the Common Method.*

```
        1.1666 Quarter girt.
        1.1666
        ------
         69996
        69996
       69996
      11666
     11666
     ----------
     1.36095556
          31.7 Length.
     ----------
     952668892
    136095556
   408286668
  -------------
  F. 43.14 291152
        12
     -------
  I. 1.50
```

Anſ. 43 feet 1 inch and a half.

| | F. I. P. |
|---|---|
| By the former method | 59—11—2 |
| By the common method | 43—1—6 |
| | 16—9—8 |

Hence it appears, that the ſeller will loſe 16 feet, 9 inches, 8 twelfths of timber, by ſelling it by the common method of meaſuring.

But what is, perhaps, ſtill more ſurpriſing is, that if the tree were cut aſunder at the line GH, the two pieces would meaſure more than when whole, even tho' the content were found by the firſt method.

Let us, for Example, ſuppoſe the tree cut aſunder at the line GH, then will the length of the butt be 12 feet 6 inches, and the quarter of the girt 1 foot 10 inches. The other part will be 19 feet 2 inches and a half nearly, and the quarter of the girt at CD, now the middle of the piece, 1 foot.

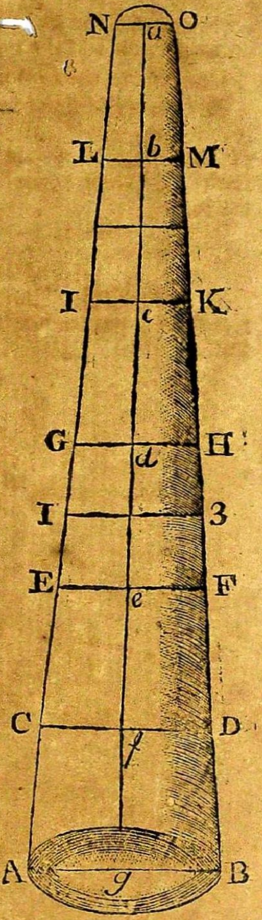
The

| | F. I. P. |
|---|---|
| The ſolid content of the butt | 42—0--2 |
| Of the top, or upper part | 19--2--4 |
| | 61--2--6 |

By this method the ſolid content of the tree is 61 feet, 2 inches, 6 twelfths.

### Rule II.

Never let the tops of your trees remain farther than they run about 7 or 8 inches the ſide of the ſquare, for this will occaſion the girting place nearer the top, and conſequently make the product leſs; nor will the addition of the length make it ſufficient to ſupply the deficiency of the girt.




To illustrate this Rule, I have drawn the figure in the margin, where the whole length of the tree is divided into six equal parts, the tree runs regularly tapering from the bottom to the top, and consequently the girts are in arithmetical progression. The length is 14 feet, and the quarters of the respective girts as follows:

That at the base

| | | F. I. P. |
|---|---|---|
| is | | 1--8--0 |
| | *f* D | 1--5--6 |
| | *e* F | 1--3--0 |
| The middle | *d* H | 1--1--6 |
| | *c* K | 0-11--0 |
| | *l* M | 0--8--6 |
| | *a* O | 0--6--0 |

Suppose it were required to find the solid content of this piece of timber.

It will be 17 feet, 8 inches, 7 parts.

Let us now suppose the piece LMNO, 2 feet 4 inches in length, cut off, then will the length be 11 feet 8 inches, the middle of the piece, the line 1--3, whose half or quarter of the girt will be 1 foot 2 inches, 9 parts.

| | F. I. |
|---|---|
| Then will the content be | 17--4 |
| To which add the content of LMNO, | 1--0 |
| Content of the two pieces. | 18--4 |

*N. B.* If the ſide of the ſquare be leſs than ſix inches, it is not eſteemed timber. But if any of the limbs are of that ſize, they are accounted timber meaſured as ſuch, and their contents added to that of the body of the tree.

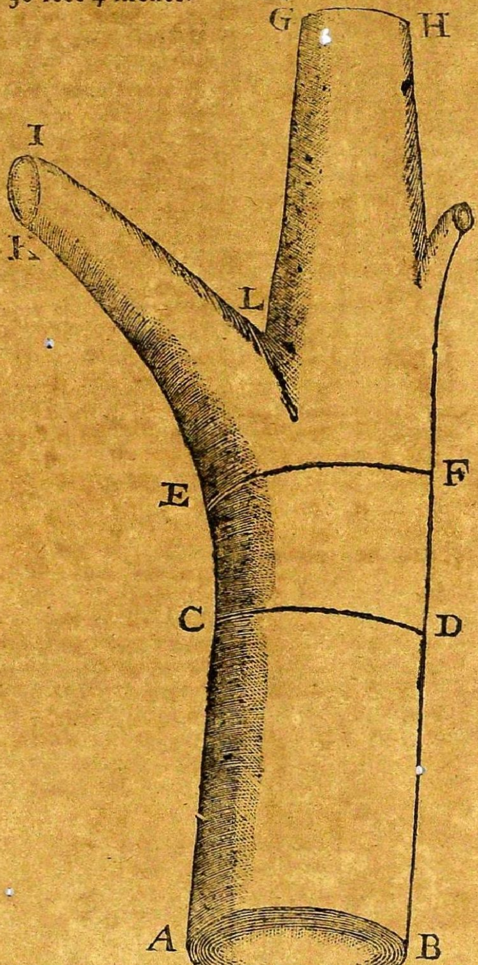
## Rule III.

It is a privilege enjoyed by the buyer, to take the girt in any part of the tree between the middle and the butt.

Exam-

EXAMPLE.

Let the figure in the margin represent a tree, whose length is 30 feet 4 inches.



| | | F. I. |
|---|---|---|
| The quarter at the butt | A B | 1--9 |
| | at C D | 1--8 |
| | at E F | 1--9 |
| | at G H | 0-10 |
| | at I K | 0—7 |
| And the length of LM, of the branch IKS | | 9—4 |

And let it be required to find the content of the whole.

As the quarter girt at E F, the middle of the tree, is greater than that of C D, the buyer will, doubtlefs, girt the tree there.

| | F. I. |
|---|---|
| Content of the tree, exclufive of the branch | 84--2 |
| ———— of the branch | 3--2 |
| | 87—4 |

Anf. 87 feet, 4 inches.

But you fhould remember, that fo large a branch growing in the manner reprefented in the figure, renders the ftick of timber very valuable to Ship-wrights, as a ufeful piece, which they call a knee, may be cut out of this tree. This obfervation is worth regarding, for I have more than once feen very fine timber of this kind fold for much lefs than its real value, through the ignorance of the grower.

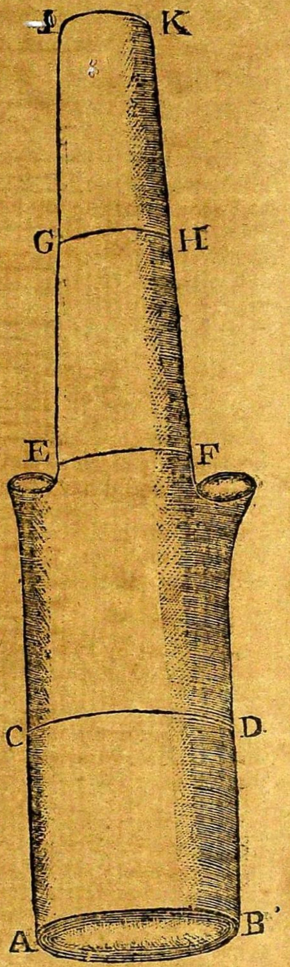
## RULE IV.

When a piece of timber does not run tapering with any regularity, it will be adviſeable to cut it into two or three pieces, unlefs the buyer will confent to the meafuring it in the manner it muft be meafured when divided into different pieces, and which practical meafurers call *joggling* the piece.

EXAMPLE.

Let the figure in the margin represent a tree, whose length from the butt to the top is 12 feet, viz. from A to F 6 feet, and from F to I 6 feet; the quarter of the girt at the butt AB, and at CD, is 1 foot 6 inches; at EF, the middle of the piece 10 inches, at GH 8 inches and a half, and at IK 7 inches.

The very inſpection of the figure, ſhews, that it would be very unjuſt to the ſeller, ſhould the girt be taken at EF, the middle of the piece; and equally unjuſt to the buyer, if taken any where between the two knots at E and F. The beſt way therefore is to conſider the ſtick of timber as compoſed of two pieces, AEFB, and EIKF. By this means juſtice will be done to both parties. Let us, therefore, ſee how many ſolid feet of timber it will contain, when properly meaſured.




First,

First, for the butt or lower part AEFB.

```
 1--6 Quarter girt.
 1--6
 ----
 9--0
 1--6
 ------
 2--3--0
       6 Length.
 -------
13--6--0
```

Second, for the top part. EIKF.

```
     0—8
       0—8 Quarter girt.
     -------
       5—4
         6 Length.
     -------
     2—8—0 Content.
Butt 13—6—0
Top   2—8—0
     -------
     16—2—0
```

Had this piece of timber been meafured at once, its content would have been as follows:

```
0—10 Quarter girt.
  0—10
  ----
  8—4
   12 Length.
 ------
 8—4—0
```

| | |
|---|---|
| Content by two dimenfions | 16—2—0 |
| Ditto when meafured at once | 8—4—0 |
| | 7—10—0 |

Confequently the feller would loofe 7 feet 10 inches of timber by meafuring it at once.

## RULE IV.

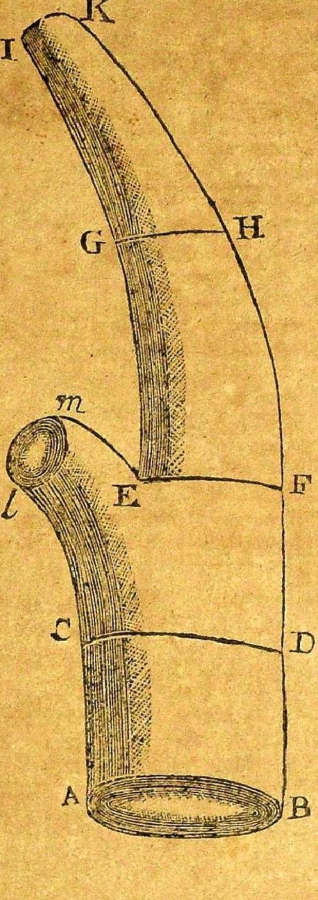
When the tree is crooked the length muft not be taken by meafuring either along the concave or convex fide: but as near as you can along the middle; for the length taken on the convex fide will be too great, and that meafured on the concave too fmall.

EXAMPLE

## EXAMPLE.

Suppose a tree were to be measured, in the form of that in the figure below; the dimensions of which are as follows: the $\frac{1}{4}$ of the girt at AB and CD, 4 feet 1 inch; at EF, 2 feet 3 inches; at GH, 1 foot 9 inches; at IK 10 $\frac{1}{2}$ inches, and that of the branch at *l m*, 1 foot 2 inches 1 twelfth: the length from B to F 7 feet, and from EF to IK, (measured along the middle of the piece) 7 feet, and of *m* F 2 feet 1 inch. What will be the content, and what will it amount to at 1 shilling and 9 pence per foot?

This piece being ſo very different in its dimenſions, muſt be ſuppoſed to be cut aſunder in the line EF, and the content of each piece added together; for that ſum will be the content of the piece.




*Solution*

*Solution by Cross Multiplication.*

First for the butt, or lower end of the piece.

```
      4--1 Quarter girt at CD.
      4--1
   ---------
      4--1
   16--4
   ---------
   16--8--1
          7 Length.
   ---------
  116--8--7 Content.
```

Second, for the top, or upper part.

```
    1--9 Quarter girt at GH.
    1--9
   ---------
    1--3--9
    1--9
   ---------
    3--0--9
          7 Length.
   ---------
   21--5--3 Content.
```

Third, for the content of the branch *lm* E.

```
   1--2--1 Quarter girt.
         1--2--1
   -----------------
         1--2--1
      2--4--2
   1--2--1
   -----------------
   1--4--6--4--1
                2--1 Length.
   -----------------
     1--4--6--4--1
   2--9--0--8--2
   -----------------
   2-10--5--2--6--1 Content.
```

| | F. | I. | P. |
|---|---|---|---|
| Content of the butt | 116 | 8 | 7 |
| ——— of the top | 21 | 5 | 3 |
| ——— of the branch | 2 | 10 | 5 |
| | 141 | 0 | 3 |

| | |
|---|---|
| 141 Feet 1s. 9d. per foot is | 12--6--9 |
| 1 Quarter or 3 twelfths of an inch is | 0--0--0¼ |
| | £. 12—6—9¼ |

Anf. The content of the tree is 141 feet, 0 inches, 3 parts, and the amount 12*l.* 6*s.* 9*d.* ¼.

It may not, my dear Difcipulus, be unneceffary to obferve, that it would be to the advantage of the feller actually to cut the tree afunder in the line EF; becaufe the butt, together with the limb, or branch, will form, what the fhipwrights call a knee, which, therefore, is more valuable than timber of the common form. This I have already told you, but thought proper to repeat it, left it fhould have flipt your memory.

*Difcip.* I remember it very well, and was juft going to make the fame remark. But at the fame time I fhould be glad to know why this fort of timber is fo valuable.

*Philo.* Its value, is, in a great meafure, owing to its fcarcity, there being few woods that abound in this kind of timber; nor is the neceffary care taken to propagate it.

*Difcip.* Is it poffible to propagate this fort of timber?

*Philo.* It is: at leaft, in fome meafure: we find, at prefent, upon heaths and other uncultivated places, a few ftraggling trees of oak. Thefe feldom grow tall or regular; they were not properly defended from the injuries of cattle, and therefore browfed upon, and ftunted while young, and thence become crooked, fhort-trunked, or pollard-trees. They are however, of great ufe to fhipwrights, and fetch a good price at the markets.

As thefe trees, though in this refpect far more valuable than tall ftrait timber, are taken very little care of, it is to be feared they will foon be all deftroyed; it is, therefore, a very ufeful attempt to difcover a regular method of producing others of a fimilar kind, and this may be eafily done by following the fame method which produced the former. They wholly owe their figure to the cattle's biting off their tops when young, and afterwards biting off the fhoots from the firft

P wound.

wound. If, therefore, a number of young trees ſet a part for the experiment, have their tops cut off at two, four, ſix, eight, ten, or twelve feet from the ground; and four years after, the ſhoots of theſe ſtunted trees be again cut in the ſame manner, they will grow up in all the irregular crooked figures that can be imagined; and, by purſuing this method, a ſupply of naturally crooked timber may be raiſed for the uſe of ſhipwrights with eaſe and certainty.

*Diſcip.* If I am not miſtaken, I have heard ſome meaſurers mention the term *meeting* of a parcel of timber; I ſhould be glad to know what they meant by that expreſſion?

*Philo.* In order to underſtand this term, which is almoſt peculiar to the large yards of ſhipwrights, it muſt be remembered, that large timber is of much greater value than ſmall. When, therefore, a quantity of timber is properly meaſured, they add the content of the ſeveral pieces together, and call that total the *meeting*. This being done, they conſider what is the value of a foot of this lott of timber, one piece with another, and according to that value the whole quantity is ſold.

*Diſcip.* Is this method juſt, both with regard to the buyer and ſeller?

*Philo.* No, certainly: a perſon not uſed to gueſs at the mean value of a parcel of timber, may be eaſily deceived by one of more experience.

*Diſcip.* Is there then no certain method by which a perſon may know the value of timber?

*Philo.* Doubtleſs there is: but a little experience is neceſſary, to know the worth of a ſingle piece of timber. When this is known, let every tree be valued ſeparately, and the ſeveral reſults added into one total. By this means, the danger of being deceived is avoided, and the work performed, with very little more trouble than by the other method.

SECT.

## SECT. III.

### *Of the Menſuration of ſtanding Timber.*

*Diſcip.* Is it poſſible to meaſure timber while it is growing in the woods?

*Philo.* It may be done: but not without great trouble and fatigue, if accuracy be required. This, however, is rarely the caſe, unleſs the price of a tree that ſtands by itſelf, or one that is of remarkable value, be deſired: an eſtimate only being thought ſufficient for this purpoſe.

*Diſcip.* How am I to proceed in finding the ſolid content of a tree before it is cut down?

*Philo.* The only method of doing this is, to climb the tree, or get up it by means of a ladder; becauſe it will be neceſſary to find the place where it meaſures only 24 inches in girt, after the proper allowance is made for the bark. Some authors, indeed, tell you, that it will be ſufficient to meaſure the girt of the tree at the middle height to which it runs timber. This would, indeed, be ſufficient, could the middle height be known: but as this cannot be diſcovered till the whole height be found, recourſe muſt be had to the firſt method. When the place where the tree is no more than ſix inches the ſide of the ſquare is found, the height or length of that part of the tree is eaſily known by the help of a line, having a plummet faſtened to it, which you may eaſily let fall till it touch the ground, and then by tying a knot in the line, where it touches the upper girting place, you may meaſure the length between the lower end of the plummet and the knot in the line after you have deſcended from the tree, and that will be the length required. When you have thus obtained the quarter girt at the top, and the length of the tree, you muſt take the girt of the tree at the bottom, add the two girts together, (allowance being made for the bark) and take the half, which will be the mean girt, or that at the middle of the tree; and

and consequently the solidity may be easily found by the methods already laid down for computing the content of tapering timber.

*Discip.* I can easily conceive this method; but the difficulty consists in putting it in practice. It will be both a tedious and a dangerous undertaking to measure the trees even in a small wood, and almost impracticable in a large one.

*Philo.* You are very right, Discipulus; and, therefore, this method is rarely practised. An estimate is generally thought sufficient; and this may be taken, with much less trouble.

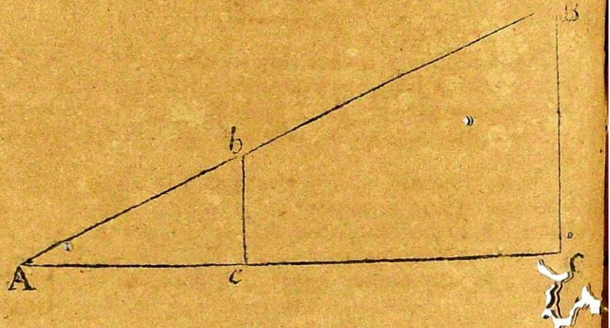
*Discip.* What are the methods most proper to be taken in forming estimates of standing timber?

*Philo.* Most of the methods commonly practised require the height of the tree, as far as it is timber, to be known; and, therefore, it will be necessary to shew you, in the first place, how this height may be found. In order to this, it will be necessary to explain a proposition or two relating to similar triangles.

*Discip.* What do you mean by similar triangles?

*Philo.* By similar triangles is meant, two, or more triangles, which have the same angles, and, consequently, their sides proportionable to one another.

Thus the two triangles in the annexed figure ABC, and A*bc*, are similar. For the angle at A, is common to both triangles, and the angle ABC is equal



to the angle A$bc$. Therefore, as A$c$ : AC : : $bc$ : BC. The ſame proportion ſubſiſts between all the other parts of the two triangles.

If, therefore, we can, by any method, meaſure the lengths of the lines A$c$, A C, and $bc$, we can find the length of the line B C, by calculation, without the trouble of actually meaſuring it.

## EXAMPLE.

Suppoſe the line A$c$ be 23.5 feet, AC 58, and $bc$ 11.5, what will be the length of B C?

## SOLUTION.

23.5 : 58 : : 11.5 : 28.38

```
           58
          ----
          920
          575
         ----
    23.5|667.0|28.38
          470
          -------
          1970
          1880
          -------
           900
           705
          -------
          1950
          1880
          -------
            70
          -------
```

Anſ. 28.38 feet.

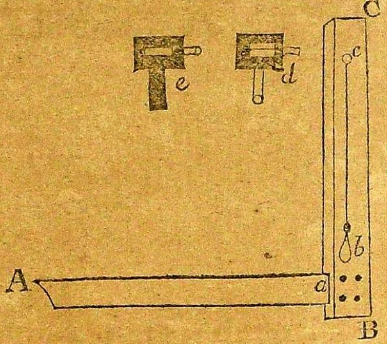
*Diſcip*. This is a very eaſy method, and I can eaſily perceive that BC may repreſent a tree, whoſe height is required; but how ſhall I find the lengths of A $c$, AC, and $bc$; and alſo know whether the two triangles are ſimilar?

*Philo*. The difficulties contained in theſe queſtions I was juſt going to remove. In order to thi , it may be neceſſary to obſerve, that AC repreſents the ground, which may eaſily be meaſured; BC the height of any object, as a tree, &c. and $bc$, a ſtaff, of any known length,

length, erected in a perpendicular poſition on the ground. Now if we can, by any method, find when the point A, the top of the ſtaff *b*, and the point B, are in a right line, the problem may be eaſily ſolved.

*Diſcip.* Thus far I underſtand the problem, and ſhould be glad to know how this grand difficulty may be removed.

*Philo.* I will endeavour to give you ſatisfaction by deſcribing a plain inſtrument, eaſily procured, and as eaſily applied to practice.

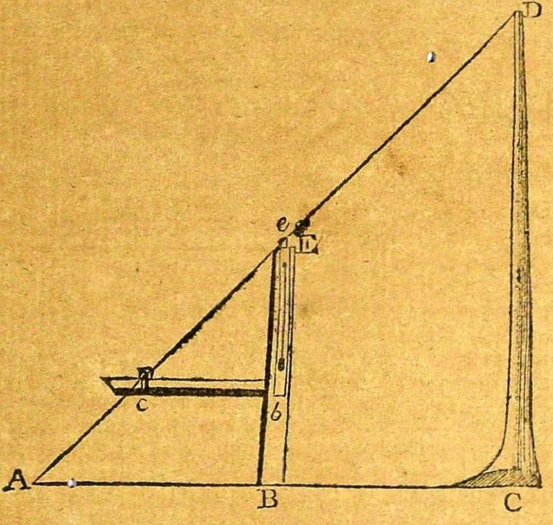



This is no other than a ſquare made like that uſed by carpenters, but larger. The figure ABC, in the margin, will convey a ſufficient idea of it. To this ſquare belong, two vanes or ſights, like thoſe on the common Gunter's quadrant. The object vane *d*, is to be fixed in a hole made in the middle of the perpendicular limb of the ſquare at C; and the ſight vane *e* muſt ſlide, by means of a braſs collet on the horizontal limb A *a*. To one of the ſides of each of theſe vanes is fixed a ſmall piece of braſs, having a little hole at the extremity, ſufficient for a ſilk thread to paſs. The thread is fixed in the hole of the braſs piece belonging to the object vane *d*, and inſerted thro' the hole in the other. On the perpendicular limb a line and plummet *b c*, is hung from a pin at *c*, in order to keep it in a perpendicular ſituation. By the help of this ſimple inſtrument, the height of a tree, or any other object may be found in the following manner:

Suppoſe

Suppoſe it were required to find the height of the tree CD.

Erect your ſtaff B E in ſome convenient place where the ground is level; fix your ſquare *b c* upon your ſtaff, in a perpendicular direction, by means of the plumb-line. This being done, and the ſtaff firmly fixed in the ground, apply your eye to the ſight-vane *c*, ſliding it forward and backward, till you perceive the hair in the object vane at *e* cut the tree in point D. Fix the vane in that place, and extend the ſilk thread till it cuts the ground as at A. Meaſure, very accurately, the length of the line AB, AC, and the height of the ſtaff, &c. B*e*. Then it will be as AB : B*e* : : A C : C D.




EXAMPLE.

Suppoſe the line A B the diſtance between the center of the ſtaff and the place where the ſilk thread touched the ground) be 25.6 feet; the height of the ſtaff,

Be 26.5, and the line AC (the distance between the point where the silk thread touched the ground, and the foot of the tree) be 51 feet: what is the height of the tree CD?

OPERATION.

As 25.6 : 26.5 : : 51 : 52.79.

```
          51
         ---
         265
        1325
        ----
25.6|1351.5|52.79
        280
       ----
       ..715
        512
       ----
        2030
        1792
        ----
        .2380
         2304
        -----
         ..76
        -----
```

Anſ. 52.79 feet, or 52 feet 9 inches nearly.

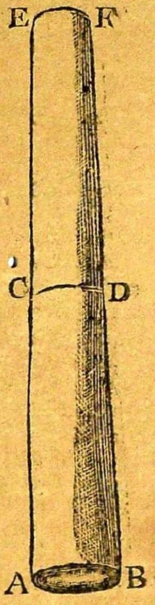
From this operation you may obſerve, that if the height of the ſtaff had been equal to the diſtance AB, the height of the object would have been equal to the diſtance AC. And as this point may be found by a few trials, you may, when you pleaſe, make uſe of this method.

Having thus found the height of the tree, you muſt aſcend the tree to the height of about 26 feet, and there take the girt of it; whence the ſolid content of it is eaſily found, by the method and rules already laid down.

*Diſcip.* You have indeed entirely removed the difficulty; but I ſtill think the meaſuring ſtanding timber a very tedious and troubleſome taſk. Is there no method of ſhortening the work, and yet forming an eſtimate near enough to anſwer the intended purpoſe?

*Philo.*

*Philo* The work may be greatly ſhortened in woods, where the trees are, in general, tall and ſtrait, and taper equally from the bottom to the top. For if you find the height of one, you may gueſs at the reſt, as they will be nearly proportional to the ſquares of their diameters.




The heights, and alſo the contents, of ſuch tapering trees may be found by girting the tree at the bottom, adding to it 24 inches, the limit for timber, and taking the half of that ſum for the true girt; for this will fall near the middle of the tree. When, therefore, this middle girt is found, and you aſcend the tree, till you find the circumference equal to it; you may conclude, that you are near the middle of the tree. The figure in the margin will give you an adequate idea of this; where ACEFDB, repreſents the body of a tapering tree; the quarter of the girt at the butt AB is nine inches, and at the top EF, ſix inches. The ſum of theſe two is 15 inches, whoſe half is 7.5; the quarter of the girt at CD. And this will always be the caſe in trees, that taper regularly from the bottom to the top.

If the heights are nearly the ſame, a very good eſtimate may be made by only taking the girt at the bottom, adding 24 inches to it, and taking the half of that ſum as the true girt; for the ſquare of one quarter of this girt multiplied by the length of a ſimilar tree, found by the method already laid down, will give the content.

If the trees be of an unequal height but equally tapering, you may find the length near enough for practice by this proportion: as the girt at the bottom of one tree is to its length; ſo is the girt at the bottom of another to its length.

In this manner yon may proceed to form an eſtimate of the whole content, and conſequently the value of any number of trees that taper equally. When they do not taper equally, recourſe muſt be had to actual menſuration by ſome of the methods already laid down; for the above proportion will not hold good in theſe caſes.

## CHAP. III.

### *Of meaſuring the tonnage of ſhips; or finding how much a ſhip will carry.*

*Diſcip.* IS it not very difficult to find the tonnage of a ſhip? for if I am not greatly miſtaken, her hold is a very irregular figure.

*Philo.* You are very right Diſcipulus; it would afford conſiderable employment for an able mathematician to ſolve the queſtion accurately; but the practical method uſed by ſhipwrights is very eaſy and conciſe.

*Diſcip.* Is it cuſtomary for ſhipwrights to find the tonnage of ſhips?

*Philo.* It is: They generally undertake the building of them at ſo much per ton; and therefore it becomes abſolutely neceſſary for them to find the tonnage.

*Diſcip.* What is the method they uſe.

*Philo.* Their method is this: they meaſure the length of the keel, the extreme breadth of the ſhip within-board, along the midſhip beam, and the depth of the hold from the plank joining the keelſon upwards to the main-deck at the midſhip beam. Theſe dimenſions being taken, the tonnage is found by the following

### RULE.

Multiply the length of the keel by the extreme breadth, and that product by the depth; the laſt product divided by 95 will give the burden in tons.

EXAMPLE

## EXAMPLE I.

Suppoſe the length of a ſhip's keel be 60 feet; the extreme breadth 30 feet; and the depth 20 feet; what is her burthen or tonnage?

## OPERATION.

```
      60
      30
    ----
    1800
      20
   -----
95)3600(37.894
   285
   ----
    750
    665
    ---
     850
     760
     ---
      900
      855
      ---
       450
       380
       ---
        70
```

Anſ. 37.894, or 38 tons nearly.

## EXAMPLE II.

A captain contracts with a ſhipwright for building a ſhip of the following dimenſions: the length of the keel is to be 112 feet 6 inches, the extreme breadth 60 feet 3 inches, and the depth 50 feet 9 inches: what will be the value of the ſhip at 5*l.* 10*s.* per ton?

OPERATION.

```
                 112.5
                 60.25
                 -----
                  5625
                 2250
                67500
               --------
               6778.125
                  50.75
               --------
               33890625
              47446875
             33890625
            -----------
95|343989.84375|3620.94972 Tons.
   285........
   ---
   .589
    570
    ---
    .198
     190
     ---
     ..858
       855
       ---
        .434
         380
         ---
         .543
          475
          ---
          .687
           665
           ---
           .225
           .190
           ----
            .35
            ---
          362094572
                5.5 Price per ton.
         ----------
         1810472860
        1810472860
        -----------
        19915.201460
```

*Anſ.* The burden is 3620.94572 tons, and the amount 19915*l.* 4*s.* 0*d.*

*Diſcip.*

*Discip.* Is this method true, and the result accurate and just?

*Philo.* It is not: the solid content in feet, resulting from multiplying the length, breadth, and depth together, is much too large: but being divided by a number also much too large, the quotient, which gives the tonnage of the ship, is tolerably near the truth.

*Discip.* That is, if I understand you right, one error is corrected by another. Would it not then be better to form another rule from undoubted principles, that would give the same result without having recourse to error?

*Philo.* Certainly it would: But this is impossible; there can no general rule be formed, that may be applied to ships in general. Nor can the hold of a ship be measured but by piece-meal, and even the solid content of several of these parts can only be found by approximation.

*Discip.* Why can no general rule be formed for finding the tonnage of all kinds of ships?

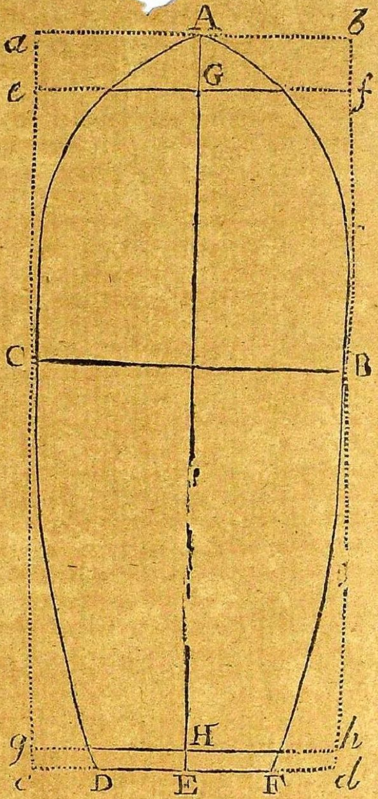
*Philo.* Because the curves, which form the bottoms of ships, are, at once, very irregular, and very different. Consequently, two ships may be of the same dimensions, with regard to length, breadth, and depth, and yet one of them carry many more tons than the other.

*Discip.* I am greatly obliged to you, Sir, for this explanation: but you said, that the result of the length, breadth, and depth, multiplied into one another, gave the solid content in feet, much too large; and that, by dividing the last product by a number also much too large, the error was, in some measure, corrected. I should be glad if you would be a little more explicit with regard to these assertions; for though you have convinced me that it is impossible to form any general rule that will give the true solid content of ships' holds in general, yet I should be glad to know whence these errors arise, and why they, in some measure, correct each other.

*Philo.* Your curiosity is highly laudable, and I will endeavour to give you all the satisfaction in my power.

 In

In order to this, let the curve superficies ABCDEF, represent the deck of a ship; then will CB be the extreme breadth, AE the length, and GH the length of the keel. But if GH be multiplied by CB, you will have the area of the parallelogram *efgh*, instead of the area of the curve superficies. The difference will be still greater with regard to depth, because the bottom of a ship is entirely formed of curve lines; but the solidity found by the last multiplication is that of a parallelopiped, in which the entire hold of the ship, supposing it every where equal in breadth with the deck, is contained; consequently the solidity resulting from the length, breadth, and depth into one another is much greater than that of the hold of a ship.




But this solidity is divided by 95, which greatly reduces the quantity of tonnage. You will be easily convinced of this when you are informed that it has been found by a variety of experiments, that a cubic foot of rain water weighs 62.5 pounds. If therefore

$2240 (= 112 \times 20)$ the number of pounds in a ton, be divided by 62.5, we ſhall have 35.8 for a diviſor, by which the content found above, ſuppoſing it right, ſhould have been divided. But 95, the common diviſor, is conſiderably more than double 35.8. Conſequently, more than half the content is allowed for the difference between the form of a ſhip's hold, and that of a parallelopiped.

*Diſcip.* But will this diviſor balance the former error; and can there be any reaſon given why the number 95 was pitched upon, rather than any other?

*Philo.* The diviſor is not ſufficient to balance the error; nor can there be any reaſon given why the number 95 was choſen; it was, probably, introduced at a time when the mathematical ſciences were little cultivated, and the principles of Hydroſtatics hardly known; and will, in all probability, be continued till ſome more exact number be found out and eſtabliſhed by authority. But though I can give you no ſatisfaction with regard to the latter part of your queſtion, I will point out a method by which the weight any ſhip, bark, or boat can conveniently carry, may be eſtimated very near the truth.

*Diſcip.* I ſhall eſteem it a very particular favour, as the ſubject is curious, and may, perhaps, be uſeful.

*Philo.* It is, undoubtedly, a curious enquiry, and, therefore, merits attention, even though it ſhould never be practiſed.

It is evident, by the laws of Hydroſtatics, that the weight of a floating body is equal to as much water as is diſplaced by it; conſequently, an entire ſhip with all her loading, &c. is equal to as much water as will form a ſolid in all reſpects ſimilar to that part of the ſhip, which is below its ſurface; or to that bulk of water that is diſplaced by the ſhip.

Hence it is evident, that the difference between the bulks of water, diſplaced by the ſhip when light and when loaded, will be the true tonnage of the ſhip. In order to diſcover what this bulk of water is, let the line, formed by the ſurface of the water when the ſhip is light, be marked on the head and ſtern: and

alſo,

alſo, another when ſhe is ſufficiently laden. This being done, let a parallelopiped be formed proportional in length to that of the ſhip's keel; in breadth, to that of the extreme breadth of the ſhip; and in depth, to that of the deep load mark. Find exactly the ſolid content of this parallelopiped, and alſo weigh it accurately in a balance. Faſhion the parallelopiped as nearly as poſſible into the form of the ſhip's bottom from the keel to the deep load-mark, and weigh it again very carefully. When this is performed, the following proportion will ſolve one part of the queſtion. As the firſt weight of the parallelopiped is to its content; ſo is the weight of it after being faſhioned to its content. This reſult, divided by 35.8, the number of ſolid feet contained in one ton of water, will give the tonnage of the ſhip, including the weight of her hull, &c.

## EXAMPLE.

Suppoſe the parallelopiped, meaſured on a ſcale proportioned to the experiment, be 168.5 feet in length, 48 feet in breadth, 20 feet in depth, and that it weighs 40 pounds 13 ounces: but when formed into a model of the ſhip, it weighs only 30 pounds 3 ounces: what is her burden?

OPERATION.

```
  168.5 Length.
     48 Breadth.
  -----
  13480
  6740
  -----
  8088.0
      20 Depth.
  --------
  161760.0 Content.

   lb  oz                       lb oz
As 40  13  :  161760  : :  30  3 : : 119647.9
   16            483         16
  ---         ------        ---
  653         485280        483
             1294080
             647040
         -------
      653|78130080|119647.9
          653.....
          ----
          1283
           653
          ----
          ·6300
           5877
           ----
           ·4230
            3918
            ----
            ·3128
             2612
             ----
             ·5060
              4471
              ----
              ·5890
               5877
               ----
               ··13
```

```
35.8|119647.9|3342.12
     1074 ...
     ------
     .1224
      1074
     ------
      .1507
       1432
     ------
       ..759
         716
     ------
         .430
          358
         -----
          .720
           719
          -----
           004
          -----
```

Anſ. 3342.12 tons; or 3342 tons, 2 hundred and 45 pounds nearly.

Let us now compute the weight of the hull of a ſhip, with her rigging, ſails, &c. that is the weight that ſinks her to her light draught of water.

In order to this, let another parallelopiped be procured, whoſe length is proportional to that of the ſhip at the light draught of water, its breadth proportional to her breadth there, and its depth proportional to the ſhip's depth from that line to her keelſon. Weigh the parallelopiped very accurately, and form it into the model of the ſhip's bottom. This being done, weigh the piece again with the ſame degree of accuracy; and the above proportion will give the content in feet, and being divided by 35.8, the tons required.

## EXAMPLE.

Suppoſe the dimenſions of the above ſhip be theſe: length 163.75; breadth 47, and depth 12.75 feet. Let alſo the weight, when rough, be 27 pounds, 6 ounces; and when faſhioned, 18 pounds, 8 ounces; and let the tonnage be required.

OPERA-

## OPERATION.

```
      163.75 Length.
          47 Breadth.
      114625
      65500
      7696.25
       12.75 Depth.
      3848125
     5387375
    1539250
    769625
    98127.1875 Content.

    lb  oz                              lb  oz
 As 27—6  :  98127.1875   : :  18—8  : 66314.6
    16              296         16
    168      5887631250         116
    27       8831446875         18
    438     1962543750          296
        438|29045647·5000|
            2628
             2765
             2628
             1376
             1314
               624
               438
              1867
              1752
               1155
                876
                2790
                2628
                 162
```

```
35.8|66314.26|1852.35
     358 . . . .
     ----
     3051
     2864
     ----
     .1874
      1790
      ----
      . . 852
          716
         ----
         1266
         1074
         ----
         .1920
          1790
          ----
           130
```

Anſ. 1852.35 tons.

| | |
|---|---|
| From the tons neceſſary to ſink the ſhip to her deep load mark. | 3342.12 |
| Take the weight of the hull, &c. | 1852.35 |
| Remains of the tons of loading the ſhip will carry. | 1489.77 |

*Diſcip.* This ſeems a far more accurate method than the former, for finding the burthen of a ſhip: but is it not difficult to ſhape the parallelopiped into the model of the ſhip?

*Philo.* It certainly is: and accordingly is hardly ever put in practice. I only mentioned it as a curious experiment, which might be uſed if the practitioner thought proper.

*Diſcip.* But is this at all applicable to ſhipwrights? can they meaſure their veſſels by this method?

*Philo.* Certainly: provided the parallelopiped could be truly formed: But as this would require the utmoſt accuracy, it is never attempted.

CHAP.

# CHAP. IV.

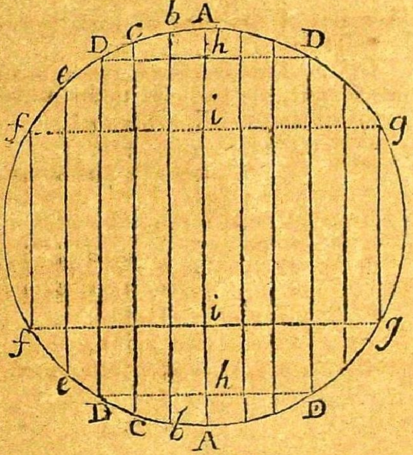
## *Of measuring Sawyer's Work.*

*Discip.* IS there any thing particular in measuring Sawyer's Work?

*Philo.* There is, if you follow what is generally called Sawyers Measure: They take the diameter of the piece that is to be cut into boards, and multiply that diameter into the length, which gives the superficial content; this result they multiply by the number of cuts, and consider the product as the true content of their work, which they are generally paid for by the hundred feet.

*Discip.* This must certainly be unjust; because neither of the cuts can be so broad, as that which goes through the center of the tree.

*Philo.* You are extremely right; and in order to demonstrate the error resulting from this method of measuring, I have drawn the figure placed in the margin, where AA is the cut whereby the whole is measured according to the method used by sawyers; and for which they are paid at London 3*s.* per hundred feet superficial measure, if the timber be oak.




If

If AA be 1 foot, then will *bb*, be 11.75 inches, *cc* 11.5, DD 10.75, *ee* 9, and *ff* 7 inches. From these dimensions let us see what will be the difference between the superficial content of the several cuts when measured truly, and when the result is found according to the practical method.

## EXAMPLE.

Suppose a piece of timber 15 feet long, and 1 foot in diameter, were to be cut into 10 boards and 2 slabs, all of an equal thickness, and the lengths of the cuts as mentioned above; what will be the superficial content of the whole, measured by the sawyers method, and what will be lost by substituting that rule instead of the true.

## OPERATION.

I. By Sawyers measure.

15 Length.
1 Cut.
———
15
11 Number of cuts.
———
165 Content.

II. By the true method.

Middle cutt, or AA.

15
1
———
15 Content in feet.
———

Second cut, or *b b*.

11.75
15
———
5875
1175
———
176.25
2 Number of equal cuts.
———
352.50 Content in inches.
———

Third

Third cut, or *cc.*

11.5
,15
575
115
17.25
2 Number of equal cuts.
345.0 Content.

Fourth cut, or DD.

10.75
15
5375
1075
161.25
2 Number of equal cuts.
322.50 Content.

Fifth cut, or *ee.*

15
9
135
2 Number of equal cuts.
270 Content.

Sixth cut, or *ff.*

15
7
105
2 Number of equal cuts:
210 Content.

Cnntent

| | | |
|---|---|---|
| Content of the 2d cuts | — | 352.5 |
| 3d ditto | — | 345 |
| 4th ditto | — | 322.5 |
| 5th ditto | — | 270 |
| 6th ditto | — | 210 |

```
12|1500 0|125
   12...
   ----
   ·30
    24
   ----
     60
     60
```

| | |
|---|---|
| Content of 2, 3, 4, 5, 6 cuts | 125 |
| of the middle cut | 15 |
| | 140 Cont. of the whole |

| | |
|---|---|
| By Sawyers meaſure | 165 |
| By the true method | 140 |
| Difference | 25 |

Anſ. The content by ſawyers meaſure is 165 feet, and the loſs by uſing that method is 25 feet.

*Diſcip*. Might not a multiplicator be found, which would give the true content by a ſingle operation?

*Philo*. Undoubtedly: and you have nothing more to do than to add the depth of the ſeveral cuts together, and multiply that ſum by the length, for the product will give the content required.

| | | I. |
|---|---|---|
| AA | —— | 12 |
| bb×2 | —— | 23.5 |
| cc×2 | —— | 23. |
| DD×2 | —— | 21.5 |
| ee×2 | —— | 18 |
| ff×2 | —— | 14 |
| | Sum | 112 |
| | Length | 15 |
| | | 560 |
| | | 112 |
| | 12 \| 1680 \| 140. | |
| | 12 | |
| | 48 | |
| | 48 | |
| | 0 | |

Anſ. 140 feet, the ſame as before.

I intended to have ſolved the queſtion in this manner, tho' you had not requeſted it; for you muſt have obſerved, that in the whole courſe of my inſtructions I have ſolved all the queſtions in the plaineſt manner, in order to avoid a fault too common in writers on theſe ſubjects, who, by endeavouring to deliver their ſolutions in a neat and conciſe manner, give the learner an infinite deal of trouble. I have, therefore, given my ſolutions in the plaineſt, though often not in the conciſeſt manner. I am willing to ſacrifice brevity to perſpicuity; and ſhall think myſelf happy if my pupils thoroughly underſtand my precepts; contractions and conciſe methods of ſolving the ſame queſtion will readily occur, when a perſon thoroughly underſtands the reaſons on which each rule is founded.

*Diſcip.* I am abundantly convinced of the truth of your obſervation; but I intended to aſk, whether any one of the cuts, multiplied by the length, would give the true content; or whether a mean multiplicator might be found, which would ſolve the queſtion.

*Philo.* It is not impoſſible, but very tedious to find a common multiplicator: an inſpection of the figure will ſufficiently inform you, that the depths of the cuts do not decreaſe in arithmetical progreſſion; for the difference between the depths of the cuts D D, and A A, is much leſs than that between *f f* and D D; notwithſtanding there are two intermediate cuts between the former, and one only between the latter. That is, the length of the line *hi*, intercepted between the dotted lines D*h*D and *fig*, is much greater than A *h*. In fact, the cuts are chords of a circle, and ſeparate parts of an unequal number of degrees. For the limb of the circle *f e*, intercepted between the cuts or chords *ff*, *e e*, is much greater than *b* A, intercepted between *b b*, and A A; notwithſtanding they are at equal diſtances, with regard to the diameter.

But though an accurate multiplicator cannot be readily found; yet if the mean cut, or that at equal diſtances between the greateſt and leaſt cut, be taken, and the length be multiplied by it, and that product multiplied by the number of cuts, the reſult will be much nearer the truth than that found by *Sawyers meaſure*; and the content found by this mean cut is what is called *market meaſure*. Let us ſee what will be the content of the above timber by this method.

```
      10.75 DD.
         15 Length.
      -----
       5375
      1075
      ------
     161.25
         11 Number of cuts.
      ------
12|1773.75|147.81 Content.
   12 . . . .
   ----
    57
    48
    ----
     93
     84
     ----
      97
      96
      ----
       15
       12
       ----
        3
       ----
```

The content found by this method is 7.81, or almost 8 feet too much; but 17.19 feet less than that found by Sawyers measure, and consequently nearer the truth by that quantity.

*Discip.* Is it not surprising, that so erroneous a method should prevail in London, where there are such numbers of persons both able and willing to correct it; especially as another method is both known and practised elsewhere?

*Philo.* It is certainly strange; but ignorance has introduced, and custom established many errors, and even absurdities, which, it is to be feared, will not soon be eradicated. I will give you another instance of this kind.

The sawyers at London have, by custom, as much per load, one piece with another, for breaking down, or cutting a piece of timber, into large scantlings, as they have for breaking down, or cutting it into small scantlings. That is, they will cut a piece of timber a foot square, of any length, into two scantlings of 12 inches by 6; or into 4 scantlings of 6 inches

inches by 6; or into 12 ſcantlings of 4 inches by 3, all at the ſame price.

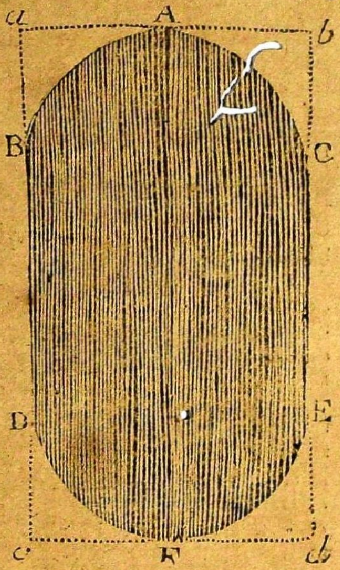
*Diſcip*. This is really aſtoniſhing! can they give any reaſon for ſo abſurd a practice?

*Philo*. They have, what they call a reaſon, for this prepoſterous method; it is this; They ſay, that where the timber proves fine it is ſeldom, perhaps never, the intereſt of the owner, to have it cut into ſmall ſcantlings. This reaſon is ſufficient to continue the practice, though they conſtantly ſee inſtances to the contrary, and are often obliged to work for whole days at half price.

## CHAP. V.

### *Of finding the content of goods for loading of ſhips, and which pay freight by the ſolid or cubic foot.*

IN order to eſtimate the freight of goods, it will be neceſſary to take the dimenſions of the ſeveral caſes, bales, truſſes, caſks, &c. in a very accurate manner. The ſhip's meaſurer is generally very careful, for otherwiſe he will loſe a conſiderable part of the freight. In order to this, the particulars ſhould be attentively examined, and if of any irregular figure, the dimenſions muſt be taken, in the largeſt parts, and paid for according to that reſult.

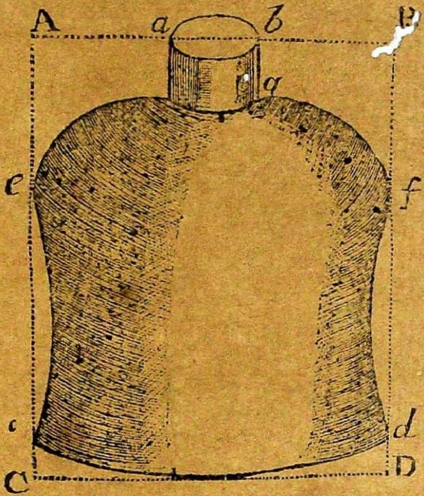



The figure in the margin will more clearly explain this remark. ABDFEC represents the plan of a case filled with goods, for which a certain price for every solid foot is to be paid as freight. But in measuring this, no regard is to be paid to the two circular ends; the case must be considered as a paralelopiped, and consequently its base as a parallelogram that is, the base as equal to *abcd*; and the area found by multiplying $ac$=AF into $cd=ab$; the same must be observed with regard to its height.

*Discip*. But is not this method of proceeding very unjust? why should any more than the real content be paid for?

*Philo*. Because no goods of any consequence can be stowed in the vacant places *a*AB, A*b*C, D*c*F, and E*d*F. They are lost to the captain of the ship, unless paid for by the owner of the goods.

*Discip*. Are all goods measured and paid for in this manner?

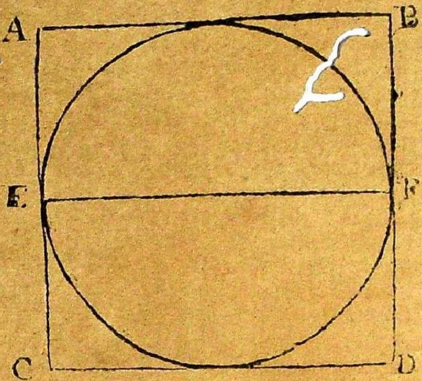



*Philo.* Yes; even large ſtills are not excepted, tho' the vacancies are much greater than in bales, caſks, &c. The figure in the margin will be ſufficient to ſhew you the method uſed in theſe caſes: *ab cdef*, repreſents the body of a ſtill, ſuppoſed to be conſigned to a perſon abroad, and to be meaſured, that the freight may be aſcertained. In order to this, the ſhip's meaſurer takes the greateſt diameter, as at *ef*=*cd*. He even meaſures round the nails, if there are any in the place where the greateſt diameter is found. This diameter is conſidered as the ſide of a ſquare, and multiplied by itſelf, which gives the ſuperficial area of the baſe, that the bottom of the ſtill is ſuppoſed to take up in the ſhip's hold; and this area multiplied by the greateſt height as A C=B D, gives what is called the ſolid content. By this means all the vacancies, as A*ae*, *b* B*f*, &c. beſides thoſe at the bottom, are taken in, and the whole paid for, as if it were a bale of cloth, &c. in the form of a parallelopiped, of the above

dimenſions.

dimensions.

The figure in the margin represents a horizontal Section or plan of the bottom of the ſtill, where E F, the diameter of the circle and equal to C D, is conſidered as the ſide of the ſquare A B C D. Let us ſee what will be the content of this ſtill, meaſured according to the above method.




### EXAMPLE.

Suppoſe C D 6.8 feet, B D (in the firſt figure) 8.5, and let it be required to find the content.

### OPERATION.

```
     6.8
     6.8
    ----
     544
    408
   -----
   46.24
     8.5
   -----
   23120
  36992
  -------
  393.040 Content.
```

*Diſcip.* I imagine, Sir, you have omitted a very material circumſtance: The body of the ſtill is not ſolid; ſhould not that particular be conſidered in the menſuration?

*Philo.* It certainly ought to be considered, were the true solidity of the still required; but the case is very different here; the question is, how much room it will take up in a ship's hold. And the ship's measurers are so very indifferent with regard to its being solid or not, that the person who sends the still is at liberty to fill it with corn, or any other goods he pleases, without any additional expence, with regard to freight.

## CHAP. VI.

### *Of measuring Cord-wood.*

*Discip.* WHAT do you mean by cord-wood?

*Philo.* By cord-wood is meant pieces of wood of different sizes, but generally pretty large, cut of the same length, and piled up very carefully, in order to be measured, and afterwards sold by the cord.

*Discip.* What do you mean by a cord of wood?

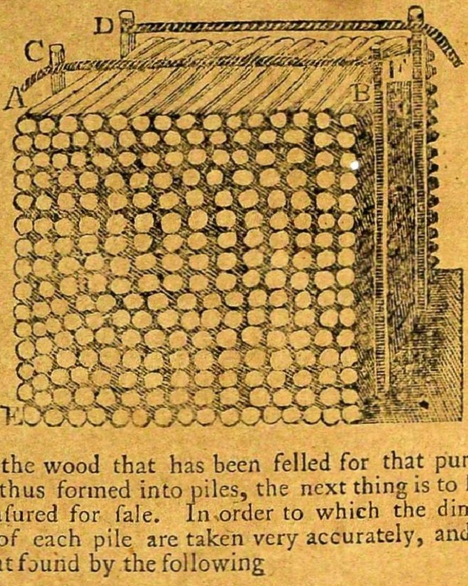
*Philo.* A cord of wood is a pile of the above pieces: but the dimensions of this pile are different, in different parts of England. What is called a statute cord is a pile eight feet long, four feet broad, and four feet high; and, consequently, its content is 128 feet; for $8 \times 4 \times 4 = 128$. This sort of cord is used in most of the northern counties of England; but in Sussex, and most of the southern counties, a pile of wood 3 feet high, 3 feet wide, and 14 feet long, is called a cord. The content of this is 2 feet less than that of the other, for $14 \times 3 \times 3 = 126$.

*Discip.* Are all the piles of these dimensions: and what methods are taken to make them so?

*Philo.* They are not all of these dimensions; for if they were, there would be no occasion to measure them. The labourers make their piles larger or smaller, according as the quantity of wood is greater or lesser. All they take care of is to cut the pieces all of the same length, or nearly so, and to pile them

up

up in a very regular manner. Accordingly, they pile the pieces up in ſo cloſe a manner, that the vacuities between them are very ſmall; and to ſupport the pile in a perpendicular ſituation, they drive four ſtakes into the ground, and faſten the tops of them with cords. The figure in the margin will ſufficiently explain this; where A E is a pile of wood ſupported in a perpendicular ſituation by the four ſtakes C D B F, faſtened together by the cords C B, D F.




All the wood that has been felled for that purpoſe being thus formed into piles, the next thing is to have it meaſured for ſale. In order to which the dimenſions of each pile are taken very accurately, and the content found by the following

## RULE.

Multiply the length by the breadth, and that product by the height: the reſult divided by 128 (or the content of a cord) will give the content required.

Suppoſe a ſtack of wood, piled up in the above manner, be 25 feet long, 18.66 wide, and 5.5 feet high; how

how many cords of wood does it contain, reckoning 128 feet to the cord?

## OPERATION.

```
         18.66 Breadth.
            25 Length.
        ------
          9330
         3753
        ------
        466.50
           5.5 Height.
        ------
        23325
       23325
      ---------
128|2565.75|20.04
    256...
    ------
    ...575
       512
       ---
       ·63
       ---
```

Anſ. 20.04 cords.

## EXAMPLE II.

Suppoſe a ſtack of wood be 30 feet 6 inches long, 20 feet 3 inches broad, and 4 feet 9 inches high; how many cords does it contain; reckoning 126 feet to the cord?

OPERA-

OPERATION.

```
            20.25 Breadth.
             30.5 Length.
            10125
           60750
           617.625
              4.75 Height.
           3088125
          4323375
         2470500
126)2933.71875(23.2834
         252
          413
          378
           357
           252
           1051
           1008
            438
            378
             507
             504
              35
```

Anſ. 23.2834 cords, or ſomething more than 23 cords and one quarter.

## CHAP. VII.

### *Of measuring Marle pits.*

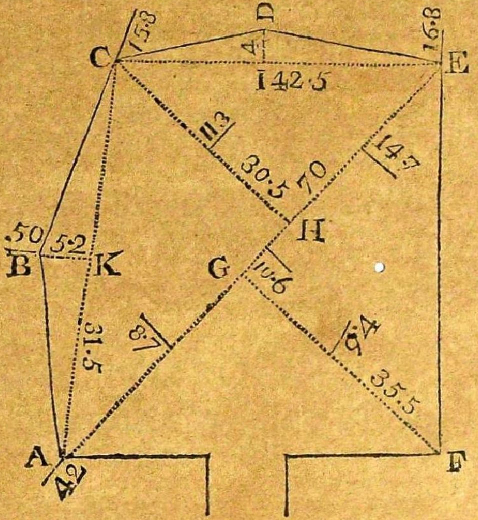
*Discip.* IS it customary to measure marle pits?

*Philo.* In many counties of England, as Lancashire, &c. where marle is found in great plenty, it is: and because I would not have you ignorant of any part of mensuration, I imagined it might not be improper to explain the method of proceeding in such cases.

Marle is known to be an excellent manure, and accordingly is used with great success by the Farmers in every county where it is found. In Lancashire, where this practice is general, they pay about twenty shillings a rod for digging and spreading the marle on the surface of the land: but the rod in that county is 8 yards, or 24 feet, which is 7.5 feet greater than the statute rod. Consequently, 64 cubic yards, or 1728 cubic feet is a rod of marle in that county. When the work is finished, the pit is measured, and the marlers paid for the quantity taken out of it. The mensuration is performed in the following manner.

They first measure the surface of the pit, and plot it, that is, draw the form of it upon paper, and divide the plot into trapeziums and triangles, as in surveying

and. This being done, they meaſure the depths in different places of the different places of the Pit, and ſet them down upon the plot, as you ſee is done in the annexed figure. This being finiſhed the content is found by the following




## Rule.

Find the ſuperficial area of the ſeveral triangles, trapeziums, &c. and multiply the area of each by its mean depth; and the ſum of all theſe, divided by the proper diviſor, will give the content of the whole pit.

## Example.

Suppoſe the content of the marle pit, repreſented in the above figure, and of the dimenſions ſet down on the reſpective lines, &c. was to be meaſured; how many ſolid rods will it contain; and what will the expence

pence amount to at nineteen ſhillings and ſix pence the rod?

*N. B.* The numbers that expreſs the depths, have lines drawn either over or under the figures.

## OPERATION.

Firſt, For the triangle C E D.

| | Depths. |
|---|---|
| C E = 42.5 | 16.8 |
| ½ D I = 2 | 15.8 |
| Area 85.0 | 2\|32.6 |
| 16.3 | 16.3 Mean depth. |
| 255 | |
| 510 | |
| 85 | |
| 1385.5 Content. | |

Second, For the triangle A C B

| | Depths. |
|---|---|
| A C = 51.5 | 15.8 |
| ½ B R = 26 | 5.0 |
| 3090 | 2.1 |
| 1030 | 3\|22.9 |
| Area 133.90 | 7.6 Mean depth. |
| 7.6 | |
| 8034 | |
| 9373 | |
| 1017.64 Content. | |

Third

Third, For the trapezium ACEF.

| | | Depths. |
|---|---|---|
| AE=70 | CH=30.5 | 16.8 |
| 33 | CF=35.5 | 15.8 |
| 210 | Sum 66.0 | 14.7 |
| 210 | ½ Sum 33. | 11.3 |
| Area 2310 | | 10.6 |
| 10.3 | | 9.4 |
| 6930 | | 8.7 |
| 23100 | | 3.4 |
| | | 2.1 |
| 23793.0 Content. | | 9\|92.8 |
| | | 10.3 Mean. |

Content of ACEF=23793
ACB= 1017.64
CED= 1385.5

1728|26196.14|15.159
1728
8916
8640
2761
1728
10334
8640
16940
15552
1388

As 1 Rod : 19*s.* 6*d.*=975*l.* : : 15.159 :
975
75795
106114
136431
14.780025
20
15.60
12
7.2

Ans.

Anſ. The content of the Pit is 15.159 rods; and the expence 14*l.* 15*s.* 7*d.*

*Diſcip.* Does not the ſurface of the marle lie at ſome diſtance below the ſurface of the ground; and if ſo, is the depth to be taken from the ſurface of the ground, or from the ſurface of the marle?

*Philo.* There are few inſtances where the upper layer or ſtratum of the marle is not below the ſurface of the ground; it is often found at about two feet below the ſurface: but whatever the depth of the earth may be, it is never conſidered in meaſuring the pit; the depths are taken from the ſurface of the marle to the bottom of the pit, without the leaſt regard to the earth, or mould above it.

## CHAP. VIII.

### *Of meaſuring Hay Ricks.*

DISCIP. Is it poſſible to find the quantity of hay contained in a rick by menſuration?

*Philo.* It is, perhaps, impoſſible to find the true content of a ſtack of hay, becauſe one rick of hay may contain more loads than another of the ſame dimenſions, according to the goodneſs of the hay, and the care taken in laying it up. But though accuracy muſt not be expected, yet a very good eſtimate may be made, and the value of a ſtack of hay in ricks computed very near the truth.

*Diſcip.* What methods muſt be taken to compute the contents of hay-ricks?

*Philo.* The dimenſions muſt be carefully taken in feet and inches, &c. and the content of the ſolid computed by ſome of the foregoing rules already laid down in this treatiſe; for this content divided by the number of ſolid feet in a load, will give the number of loads contained in the rick.

*Diſcip.* Is it known how many ſolid feet make a load of hay?

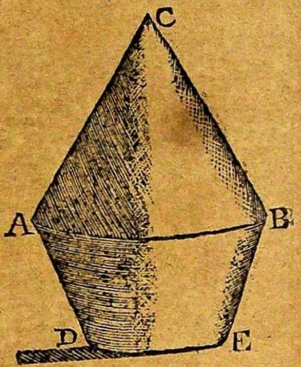
*Philo.*

*Philo.* The precife number cannot always be known; and this renders it impoffible to compute the number of loads contained in the rick. For could this number be known, the content of a hay-ftack in loads, might be computed with fufficient accuracy. A load of hay contains 36 truffes, each trufs weighing 56 pounds, or half a hundred weight; fo that 18 hundred weight is a load of hay. But the contents of the truffes are various; fome of them containing near 14 folid feet, and others little more than 7, though both truffes are of the fame weight. However, if 10 or 11 folid feet be confidered as a mean, the computation will not be far from the truth. Upon this foundation, a load of hay will contain about 360 or 400 folid feet.

When the number of folid feet in a trufs, or a load are determined, the content of the hay-ftacks muft be found; and in order to this, the form of the folid muft be confidered. There will be no great difficulty in this, as all hay-ftacks ftand either on a rectangular or a circular bafe; and are compofed of two tapering folids, whofe content will be eafily found by the rules already laid down. This will fufficiently appear, if the two forts of ricks be confidered feparately.

1. Of

## 1. *Of the circular Hay-Rick.*




The figure in the margin represents a circular hay-rick; and it is sufficiently evident from the figure, that the whole stack is composed of two tapering solids A B C, and A B D E. The upper is a cone, standing upon its proper base; and the latter, a frustum of a cone, standing upon its lesser base. But the solid content of the cone A B C, is easily found by the rule laid down page 145: and that of the frustum A B D E, by either of the rules mentioned page 157. Hence the content of the whole may be easily found. And this content divided by 360, or 400, according as the hay has been well or ill made, will give the number of loads required.

### EXAMPLE I.

Let it be required to find how many loads of hay are contained in the above rick, supposing the diameter of the thickest part of the rick A B, 30 feet 6 inches, the perpendicular height of the cone A B C 20 Feet 3 inches; the lesser diameter D E of the frustum A B D E 24 feet 9 inches, and its perpendicular height 8 feet.

1. For

1. For the upper Solid.

*Operation.* See Page 145, 146.

```
                30.5
                30.5
                ----
                1525
               9150
              ------
              930.25  Square of A B.
               .7854
              ------
              372100
             465125
            744200
           651175
          ----------
          730.618350  Area of the Bafe.
                6.75  One 3d of the Height.
          ----------
          365309175
         511432845
        438371010
        -----------
4|00)4931.6738625(12.3291846 Loads.
     4
     -
     .9
      8
      -
      13
      12
      --
      11
       8
      --
      .36
       36
      ---
      ..7
        4
        -
       33
       32
       --
       .18
        16
        --
       .26
        24
        --
       .225
```

2 For

2. For the lower Solid.

*Operation.* See *Rule* 2, Page 157.

```
           24.75
            30.5
           -----
           12375
          74250
         -------
         754.875
          10.594 Add.
         -------
         765.469
          .7854
         -------
         3061876
        3827345
       6123752
      5358283
      -----------
      601.1993526
                8
      -----------
4|00)4809.59482|08)
     12.0239870
     -----------
     12.3291846 Cone.
     12.0239870 Frustum.
     -----------
     24.3531716 Loads.
```

Ans. 24 Loads, 1 Quarter, and a Half, nearly.

## EXAMPLE II.

How many Loads of Hay are contained in a circular Hay-rick, whose largest Diameter is 60 Feet, its lesser 40; the Perpendicular Height of the Upper Solid 50 Feet, and that of the lower 20 Feet: and what will the Whole amount to at 2*l.* 5*s.* per Load?

OPERA-

OPERATION.

```
         60
         60
       ----
       3600
       .7854
      ------
      14400
     18000
    28800
   25200
   ----------
   2827.4400 Area of the Base.
      16.66  One third Height.
  -----------
  1696464
 1696464
1696464
282744
-------------
47105.1504 Content upper Solid.

        60
        40
      ----
      2400
    133.33 Add.
    -------
   2533.33
     .7854
   -------
   1013332
  1266665
 2026664
1773331
----------
1989.677382
        20
------------
39793.547640 Content lower Solid.
47105.1504 Upper Solid.
4|00)86898.698(04
217.24674 Loads.
```

As

As 1 Load 2*l.* 5*s.* (=2.25) : : 217.24674 :

```
                 2.25
            ---------
            108623370
            43449348
            43449348
            ---------
        £488.80,1650
                  20
            ---------
         s 16.103300
                  12
            ---------
         d 1.2396
```

Anſ. The Content of the Rick is 217.24674 or ſomething more than 217 Loads, and the value 488*l.* 16*s.* 1*d.*

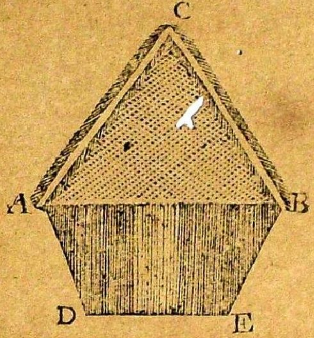
*Diſcip.* Is this the Method generally practiſed in meaſuring Hay-ſtacks of a circular Figure?

*Philo.* I do not remember to have ever ſeen any Method practiſed: experience is the general Guide, and the Contents of Hay-ſtacks are commonly gueſſed at by thoſe who have been long converſant in Huſbandry.

*Diſcip.* This muſt ſurely be a very precarious Method; and I ſhould imagine that a Stack of Hay valued by Menſuration, muſt be far nearer the Truth, than that which is only gueſſed at by the moſt experienced Farmer. But I am ſorry I have interrupted you with theſe impertinent Queſtions.

*Philo.* I am always ready to ſatisfy any doubt that may ariſe in your Mind; and will now proceed to the other Kind of Hay-ricks; that is, ſuch as ſtand upon a rectangular Baſe.

The




The Figure A B C D E, placed in the Margin, repreſents a Hay-rick ſtanding upon a rectangular Baſe; and is compoſed of two Solids: the upper A B C is a Priſm; the lower A B D E, an irregular Solid having its Baſes parallel but unequal, its two Sides equal but not parallel, and the Lengths, or diſtances, between its two Baſes alſo unequal. The Content of the upper Solid may be found by the Rule given in Page 142, and the latter eaſily reduced to a parallelopiped, and the Content found by the Method laid down, Page 132.

### Example I.

A Farmer ſells, at 35 Shillings a Load, a rectangular Rick of Hay of the following Dimenſions; the greater Baſe A B is 36 Feet, the leſſer Baſe D E 24 Feet; the greater Length, meaſured from A. and B, 80 feet, the leſſer, or that meaſured from D and E 72 Feet; the Perpendicular Height of the lower Solid A B D E 20 Feet, and that of the upper A B C 28 Feet: What will the Whole amount to

1. Upper Solid.

| | |
|---:|---|
| 36 | Baſe A B |
| 14 | Half Perp. |
| 144 | |
| 36 | |
| 504 | |
| 80 | Length. |
| 40320 | Content. |

2 Lower

## 2. Lower Solid.

In order to find the content of the lower solid ABDE, you must find a mean base, by multiplying the two bases together, and extracting the square root out of the product; a mean length must also be found in the same manner. The solid will then be reduced to a parallelopiped, and the content found by the rule, page 132.

By pursuing this method the mean base will be 29.39, and the mean length 75.9. Then

```
     29.39 Mean Base.
     29.39
     ------
     264511
      817
    26451
    5878
    ---------
   863.7721
       75.9 Mean Length.
   ---------
   77739489
  43188605
 60464047
 -----------
 65560.30239 Content.

      40320 Upper Solid.
      65560.3, &c. Lower.
4|00(10588|0.3)264.7 Loads.
      8...
      --
      25
      24
      --
      18
      16
       28
       28
       --
       ..
```

As 1 Load : 1*l.* 15*s.* (=1.75) : : 264.7 :

```
   1.75
  13235
 18529
 2647
£463.225
     20
 s. 4.500
```

Anſ. 463*l.* 4*s.* 6*d.*

## EXAMPLE II.

A gentleman purchaſes of a farmer for 370*l.* a rectangular rick of hay of the following dimenſions; the greater baſe 40 feet, the leſſer 30; the greater length 70 feet, the leſſer 60; the perpendicular height of the lower ſolid 20 feet, and of the upper 30; what did he pay per load?

1. Upper Solid.

```
  40 Baſe.
  15 Half perp.
 200
  40
 600
  70 Length.
42000
```

2. For the lower solid.

```
        34.64 Mean base.
        34.64
       ------
        13856
       20784
      13866
     10392
    ----------
    1199.9296
         64.8 Mean length.
    ----------
     959943368
    479971184
   71995776
   ------------
   77755.43808 Content.

      42000 Upper solid.
      77755.438 Under.
   4,00)119755.4|38(299.388
      8 . . . .
      --
      39
      36
      --
       37
       36
       --
        15
        12
        --
         35
         32
         --
          34
          32
          --
           2
```

Loads. £. £. s. d.

As 299.388 : 370 :: 1 : 1 : 4 : $8\frac{1}{4}$

Ans. The content of the rick is 299.388 loads, and the price per load 1*l*. 4*s*. 8*d*. $\frac{1}{4}$.

CHAP.

# CHAP. IX.

## *Of measuring bodies, whose contents cannot be found by any of the foregoing rules.*

*Discip.* ARE there any bodies whose solidity cannot be found by the common methods of mensuration?

*Philo.* Yes, several: as statues, and other pieces of sculpture, furze-faggots, &c.

*Discip.* By what method can the solid content of bodies of this kind be accurately known?

*Philo.* By a very easy method: it is this; let a vessel in the form of a parallelopiped be formed of sufficient capacity to contain the body, whose solidity is desired. Pour into this vessel as much water as will be sufficient to cover the body when placed in the vessel, and note the height of the water exactly; Immerse the body, and note how high the fluid has risen: for the solid content of the additional space, occupied by the water, on account of the immersed body, will be equal to the solidity of that body.

### EXAMPLE I.

Suppose a parallelopiped to be formed, whose length is 7 feet, and its breadth 3 feet 6 inches; and that a statue being immersed in a sufficient quantity of water in this vessel, raises the fluid 1 foot 6 inches; what is the solid content of that statue?

```
  3.5 Breadth
    7 Length.
 ----
 24.5
  1.5 Rise of the water.
 ----
 1225
 245
 ----
36.75 Solidity.
```

Anf. 36.75 feet, or 36 feet 9 inches.

## EXAMPLE II.

Suppofe a large furze-faggot was immerfed in the fame veffel, and the rife of the water was found to be 1 inch, what is the folid content?

```
   3.5 Breadth.
     7 Length.
  ----
  24.5
  .083 Rife of the water.
  ----
   735
 1960
 ------
2.0335 Content.
```

# CHAP. X.

*A table of Englifh meafures, &c.*

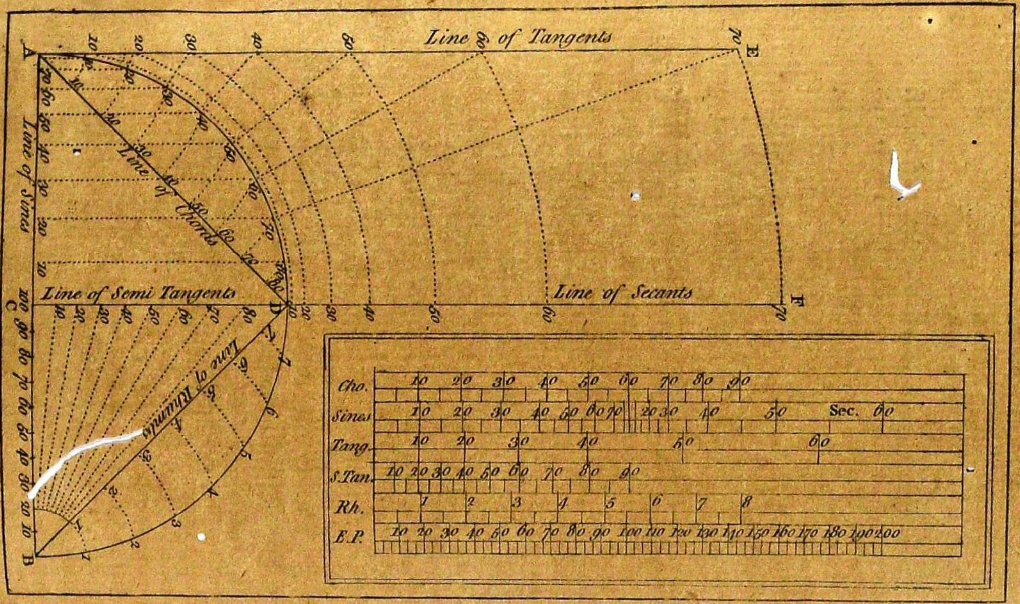
*Philo.* IT is not my intention to give you here the common table of Englifh Meafures; but fuch only as may be ufeful in Menfuration.

A Palm is 3 inches.
A Hand is 4 inches.
A Span is 9 inches.
A Square foot is 144 fquare inches.
A Cubic foot is 1728 cubic inches.
A Cubit is 1 foot 6 inches.
A Square yard is 9 fquare feet.
A Cubic yard is 27 cubic feet.

An

An Ell is 1 yard and 9 inches.
A Geometrical pace is 5 feet.
A Fathom is 6 feet.
A Statute pole, rod, or perch is 16 feet and a half.
Gunter's chain is 4 ſtatute poles, or 22 yards.
A Fen, or Woodland pole is 18 feet.
A Foreſt pole is 21 feet.
A Furlong is 40 poles, or 220 yards.
A Mile is 8 furlongs, or 1760 yards.
A Square ſtatute pole is 272.25 ſquare feet.
A Square woodland pole is 324 ſquare feet.
A Square foreſt pole is 441 feet.
A Rood is 40 ſquare poles, or 1 quarter of an acre.
An acre 160 ſquare poles.
A Load of rough timber is 40 feet.
A Load of ſquared timber is 50.
A Load of 1 inch plank is 600 ſquare feet.
A Load of 1 $\frac{1}{2}$ inch plank is 400 ſquare feet.
A Load of 2 inch plank is 300 ſquare feet.
A Load of 2 $\frac{1}{2}$ inch plank is 240 ſquare feet.
A Load of 3 inch plank is 200 ſquare feet.
A Load of 3$\frac{1}{2}$ inch plank is 170 ſquare feet.
A Load of 4 inch plank is 150 feet ſquare.
A Chaldron of Newcaſtle coal is 60 cubic feet.
A Wincheſter buſhel is 2150.4 ſolid inches.

When the buſhel is heaped up, the content is in proportion to that of the ſame buſhel when ſtruck as 4 to 3. Therefore the conical heap is one third of the cylindrical content, or 716.8 ſolid inches.


A
B
C
D
E
F
Line of Tangents
Line of Secants
Line of Semi Tangents
Line of Sines
Line of Chords
Line of Rhumbs
Cho.
Sines
Sec.
Tang.
S.Tan.
Rh.
E.P.

# CHAP. XI.

*Of the plain ſcale, the manner of conſtructing the ſeveral lines upon it, and the uſes it is generally applied to in the various branches of mathematical computation.*

*Diſcip.* WHAT do you mean by a plain ſcale?

*Philo.* A plain ſcale, is a thin piece of braſs, ivory, or wood, generally about 12 inches long, and 1 inch and a half broad; on which various lines are projected, for the menſuration of angles, heights, diſtances, &c.

*Diſcip.* Why is it called a plain ſcale?

*Philo.* Becauſe the lines projected upon it are ſuch as naturally reſult from a circle, without having recourſe to any artificial numbers, &c.

*Diſcip.* What are the lines generally placed on a plain ſcale?

*Philo.* A great variety of lines may be projected for this purpoſe, and all of them have their uſes; but the moſt general are, a line of chords, of ſines, of tangents, of ſecants, ſemi-tangents, and equal parts.

*Diſcip.* How are thoſe lines projected?

*Philo.* They are all graduated from the circle, in a very natural manner. But in order to render this uſeful part of mathematical conſtruction as plain and eaſy as poſſible, I ſhall conſider each line ſeparately, and ſhew how each of them may be projected, and conſequently, the plain ſcale completely finiſhed by the meaneſt capacity. But it will be neceſſary previouſly to obſerve, that the circumference of every circle is ſuppoſed to be divided into 360 equal parts, called degrees; and each degree into 60 equal parts called minutes; and each minute into 60 equal parts called ſeconds, &c.

1. Of

## 1. *Of the line of chords.*

The chord, or subtense of an arch, is a right line connecting the extremities of the arch together, and has been already defined, page 31. If, therefore, the several degrees of the quadrant, or quarter of a circle, be transferred to a right-line, drawn from one extremity of the quadrant to the other, that line will be a line of chords.

In order to explain this description of a line of chords, place one foot of the compasses in the center C (see the figure on page 314.) and with any convenient radius, describe the semi-circle ADB, and divide it into two equal parts or quadrants, by the line CD. Divide the arch of the quadrant into 9 equal parts, and number these divisions with 10, 20, 30, 40, &c. from A to D, as you see in the figure. Then will each of these divisions contain 10 degrees: subdivide each of these divisions into 10 equal parts; so will the whole quadrant be divided into 90 degrees. But as these subdivisions would have been so very close together in the figure annexed, as to have rendered the whole confused, they are omitted. The arch being thus divided draw the right-line AD, from A, the one extremity of the arch, to D the other. Place one foot of the compasses in A, and extend the other to 10 on the arch, and transfer that distance to 10 on the right-line: that is, keep one of the points of the compasses firm on the point A, and with the other sweep a small arch, till it touches the right-line A D. Do the same by all the rest of the divisions, 20, 30, 40, &c. Keeping one of the points still fixed on the point A. By this means the equal divisions of the arch will be transferred to the right-line, where they will become unequal; and the line AD divided in this manner, will be a line of chords.

You must not imagine that a line of chords projected from a circle of so small a radius can be of any real use in practice: The radius of the circle must be much larger,

larger, that there may be a proper ſpace between the diviſions on the line when finiſhed; otherwiſe it will be impoſſible to meaſure or conſtruct any angle with ſufficient accuracy. The foregoing figure was added merely to explain the manner of conſtructing the ſeveral lines on the plain ſcale; and not by any means to ſerve as an example with regard to its magnitude.

### 2. *Of the line of ſines.*

From the points of the ſeveral diviſions 10, 20, 30, &c. of the quadrantal arch A D, draw lines parallel to D C, till they cut the radius AC, then will the right-line CA be a line of ſines. And if we ſuppoſe the arch AD, to be divided into 90 equal parts, and that lines parallel to DC be drawn from the ſeveral points of thoſe diviſions, the line CA will be divided into a line of ſines, anſwering to every ſingle degree of the quadrant, and which muſt be numbered from C towards A.

*N. B.* If the line AC, divided as before, be numbered from A towards C, you will have a line of verſed ſines, defined page 31.

### 3. *Of the line of Tangents.*

From A, the extreme point of the arch, raiſe the perpendicular A E; and from the center C, through the ſeveral diviſions of the quadrant AD, draw lines till they cut the tangent AE (ſee page 31.) ſo then will the line AE, become a line of tangents, and muſt be numbered from A towards E. But as the diſtances between the diviſions become very large, as they approach towards 90 degrees, the tangent line is continued no farther than 70 degrees, in order to keep it within a convenient length.

### 4. *Of the line of Secants.*

The secant of any arch has been already defined, page 31. and consequently the lines drawn from the center C, through the several divisions of the quadrant AD, to divide the line of tangents, are the secants of their respective arches: If therefore, their lengths be transferred to the line C D, continued to F, the line C F will be a line of secants; and must be numbered from C towards F.

After the same manner may the lines of sines, tangents, and secants, be constructed to every minute of the quadrant, provided the radius be large enough to admit of the arches being divided into the requisite number of equal parts.

### 5. *Of the line of Semi-Tangents.*

The line of semi-tangents is so called from the lengths of the several divisions, being equal to the respective tangents of half that number of degrees. That is, the length of the semi-tangent of 90 degrees, is equal to the tangent of 45 degrees; the length of the semi-tangent of 80 is equal to the tangent of 40, &c. This being premised, the line is easily projected in the following manner: lay a ruler from the point B, to the several divisions of the quadrant AD, and mark where it cuts the line CD; for these divisions on C D will be the semi-tangents of their respective arches; and consequently the line C D will be a line of semi-tangents, and must be numbered from C towards D.

### 6. *Of the line of Rhumbs, or Rumbs.*

By the term rhumbs are meant the points of the Mariner's Compass, which is divided into 32 equal parts: consequently each quadrant of the circle contains eight of those points or rhumbs; and the interval between them is equal to 11 degrees, 15 minutes; for

11

11 degrees, 15 minutes, multiplied by 8 is equal to 90, the number of degrees contained in a quadrant of a circle. The line of rhumbs is therefore projected in the following manner:

Divide the quadrantal arch BD into 8 equal parts; and draw the right-line BD, from one extremity of the arch to the other. This being done, transfer the several divisions of the arch to the right-line: That is, place one foot of the compasses in the point B, and extend the other successively to the several divisions of the quadrant, and with these extents, draw small arches till the other foot touches the line BD: then will that line thus divided be a line of rhumbs, which must be numbered from B towards D.

*Discip.* You mentioned the mariner's compass in explaining the method of constructing the line of rhumbs; I should be glad if you would give me some account of that instrument, because otherwise I shall know very little of this line.

*Philo.* The mariner's compass is the instrument by which the ship is steered, and represents the horizon. It is divided into 32 equal parts called rhumbs or points, by lines drawn from the center to the circumference; and the outer circle into 360 degrees. Upon this part, and directly on the north and south line, is fastened a square bar of steel, which, being touched with a loadstone, acquires a certain virtue, whereby it always hangs nearly under the meridian, when the whole is suspended on the point of a needle, placed perpendicularly under the center in a box for that purpose.

### 7. *Of the line of equal Parts.*

This line is easily divided when it is determined how many of these parts the radius is to contain. If the scale be intended for the purposes of navigation, it should be divided into 60 equal parts; because 60 geographical miles makes a degree of longitude on the equator: and as the length of a degree of longitude in any parallel, is to the length of a degree of the equator,

as

as the co-sine of the latitude is to radius; therefore the length of a degree on any parallel may be easily measured by it. But as this scale is not confined to navigation, I have divided the radius, or C B, into 100 equal parts.

### *Use of the plain Scale.*

This scale is of great use in every branch of the mathematics, where there is a necessity for laying down and measuring angles, &c.

I have shewn the method of graduating a plain and diagonal scale of equal parts, page 50, 51, &c. and their uses in surveying land; and shall now shew the use of the scale just described, in projecting and measuring any of the parts of a triangle, when a sufficient number of these parts are given.

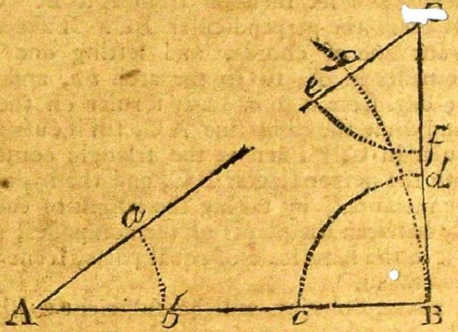
For it is necessary, before any question can be solved, that proper data be given, or found by actual mensuration; and these data are what I called the given parts of a triangle. It is impossible, for instance, to find the area of a square, unless the side, or something analogous to it, be given; the same may be said of a triangle, or any other geometrical figure.

*Discip.* You have fully explained the difficulty; and I beg you will now proceed to the mensuration of the several parts of a triangle.

*Philo.* A triangle is composed of three sides and three angles; when one of the angles is just 90 degrees, it is called a right-angled triangle; but if neither of the angles be a right one, it is called an oblique-angled triangle.

The

The figure below is a right-angled triangle; becauſe the angle at B, or A B C, which is meaſured by the arch *cd*, is equal to 90 degrees. The line AB is called




the baſe, BC the perpendicular, and AC the hypothenuſe. The angle at A is meaſured by the arch *ab*, and that at C by the arch *ef*.

It is demonſtrated by geometrical writers, that the three angles of any right-lined triangle are equal to 180 degrees. But one of the angles of a right-angled triangle is equal to 90 degrees; therefore the ſum of the other two angles is equal to 90 degrees; conſequently if one of theſe angles be given, the other will be eaſily found, by ſubſtracting the given angle from 90 degrees.

This being premiſed, it will be eaſy to project and meaſure any of the parts of a right-angled triangle, if either the baſe, perpendicular, or hypothenuſe, together with one of the oblique angles be given.

EXAMPLE.

Suppoſe the baſe AB, be 80 yards, and the angle at A 35 deg. and let it be required to project the angle BAC, and meaſure the perpendicular BC, and the hypothenuſe AC.

## SOLUTION.

Draw the right-line AB; take 80 equal parts from your ſcale and ſet them off from A to B. On the point B erect the perpendicular BC. Take 60 deg. from your line of chords, and ſetting one foot of the compaſſes in A, ſweep the arch *b a*, and ſet off on it 35 deg. from *b* to *a*. Lay a ruler on the points A, *a*, and draw the right-line A C, till it cuts the perpendicular in C. Then is the triangle conſtructed, and both the perpendicular B C, and the hypothenuſe may be meaſured, by taking the length of each ſucceſſively between the points of the compaſſes, and applying it to the ſame ſcale of equal parts, from whence AB was taken.

The above Solution will be ſufficient to ſhew the method of conſtructing a triangle, and meaſuring its ſeveral parts. It is alſo evident from the figure that if AB be conſidered as the radius, and the arch B *g* be drawn, that BC will be the tangent of the angle at A and AC its ſecant. If CB be ſuppoſed to be the radius, BA will be the tangent of the angle at C and C A its ſecant. But if A C be ſuppoſed the radius, then will AB be the ſine of the angle at C, and CB, the ſine of the angle at A.

Theſe few obſervations will be ſufficient to point out the uſes of the plain ſcale, in the doctrine of triangles, on which moſt of the mathematical ſciences depend.

If BC repreſent a tower, tree, &c. and you meaſure the baſe A B, and take the angle at A, with ſome proper inſtrument, the triangle may be eaſily projected, and the height of the object meaſured on the ſcale of equal parts, from whence AB was taken. Some examples of this are given in page 97, 98, &c.

Thus have I conducted you through every part of what is generally called menſuration; and ſhall now leave you to put theſe precepts in practice. On ſome future occaſion, perhaps, I may inſtruct you in other branches of the mathematics, in the ſame eaſy and familiar manner.

FINIS.